

## 2.

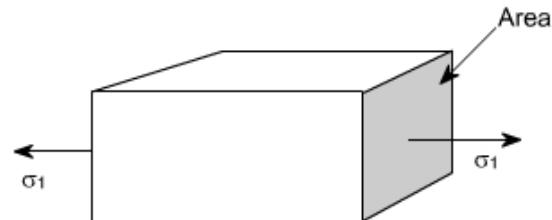
# Principal Stress and Strain

## Theory at a Glance (for IES, GATE, PSU)

### 2.1 States of stress

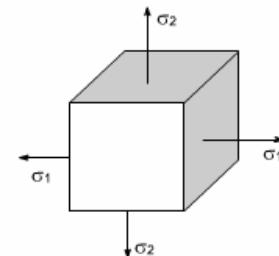
- **Uni-axial stress:** only one non-zero principal stress, i.e.  $\sigma_1$

Right side figure represents Uni-axial state of stress.



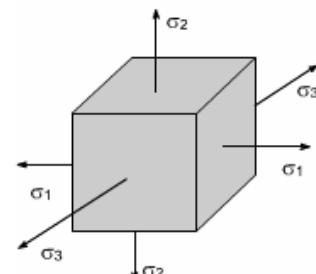
- **Bi-axial stress:** one principal stress equals zero, two do not, i.e.  $\sigma_1 > \sigma_3$  ;  $\sigma_2 = 0$

Right side figure represents Bi-axial state of stress.



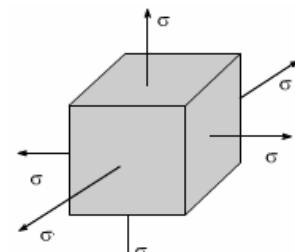
- **Tri-axial stress:** three non-zero principal stresses, i.e.  $\sigma_1 > \sigma_2 > \sigma_3$

Right side figure represents Tri-axial state of stress.



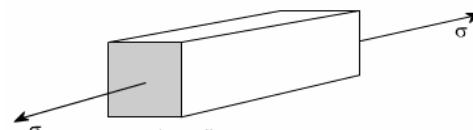
- **Isotropic stress:** three principal stresses are equal, i.e.  $\sigma_1 = \sigma_2 = \sigma_3$

Right side figure represents isotropic state of stress.



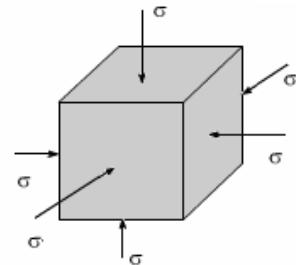
- **Axial stress:** two of three principal stresses are equal, i.e.  $\sigma_1 = \sigma_2$  or  $\sigma_2 = \sigma_3$

Right side figure represents axial state of stress.

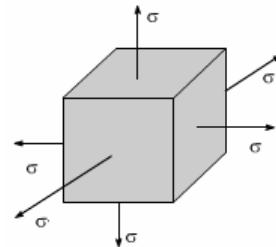


- **Hydrostatic pressure:** weight of column of fluid in interconnected pore spaces.  
 $P_{\text{hydrostatic}} = \rho_{\text{fluid}} gh$  (density, gravity, depth)
- **Hydrostatic stress:** Hydrostatic stress is used to describe a state of tensile or compressive stress equal in all directions within or external to a body. Hydrostatic stress causes a change in volume of a material. Shape of the body remains unchanged i.e. no distortion occurs in the body.

Right side figure represents Hydrostatic state of stress.



Or



## 2.2 Uni-axial stress on oblique plane

Let us consider a bar of uniform cross sectional area A under direct tensile load P giving rise to axial normal stress  $P/A$  acting on a cross section XX. Now consider another section given by the plane YY inclined at  $\theta$  with the XX. This is depicted in following three ways.

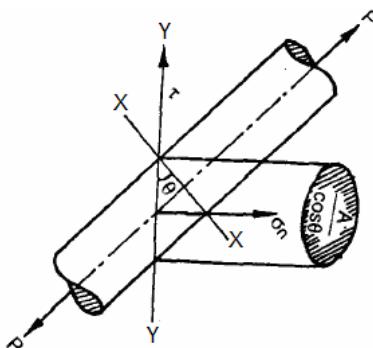


Fig. (a)

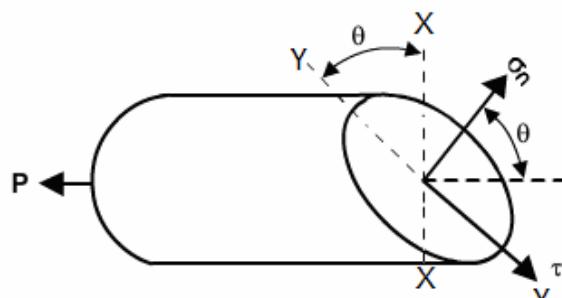


Fig. (b)

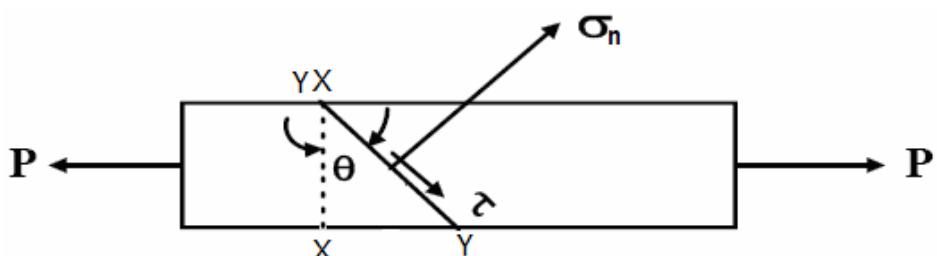


Fig. (c)

Area of the YY Plane =  $\frac{A}{\cos \theta}$ ; Let us assume the **normal stress** in the YY plane is  $\sigma_n$  and there is

a **shear stress**  $\tau$  acting parallel to the YY plane.

Now resolve the force P in two perpendicular direction one normal to the plane YY =  $P \cos \theta$  and another parallel to the plane YY =  $P \cos \theta$

## Chapter-2

## Principal Stress and Strain

Therefore equilibrium gives,

$$\sigma_n \frac{A}{\cos \theta} = P \cos \theta \quad \text{or} \quad \sigma_n = \frac{P}{A} \cos^2 \theta$$

$$\text{and } \tau \times \frac{A}{\cos \theta} = P \sin \theta \quad \text{or} \quad \tau = \frac{P}{A} \sin \theta \cos \theta \quad \text{or}$$

$$\tau = \frac{P}{2A} \sin 2\theta$$

- Note the variation of **normal stress**  $\sigma_n$  and **shear stress**  $\tau$  with the variation of  $\theta$ .

When  $\theta = 0$ , normal stress  $\sigma_n$  is maximum i.e.  $(\sigma_n)_{\max} = \frac{P}{A}$  and shear stress  $\tau = 0$ . As  $\theta$  is increased, the normal stress  $\sigma_n$  diminishes, until when  $\theta = 90^\circ$ ,  $\sigma_n = 0$ . But if angle

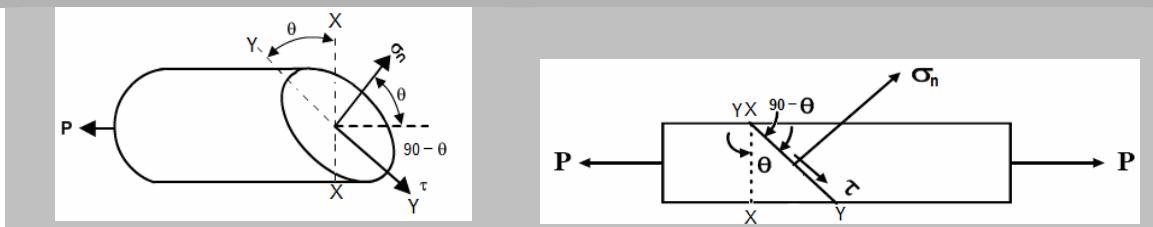
$\theta$  increased shear stress  $\tau$  increases to a maximum value  $\tau_{\max} = \frac{P}{2A}$  at  $\theta = \frac{\pi}{4} = 45^\circ$  and then diminishes to  $\tau = 0$  at  $\theta = 90^\circ$

- The shear stress will be maximum when  $\sin 2\theta = 1$  or  $\theta = 45^\circ$

- And the maximum shear stress,  $\tau_{\max} = \frac{P}{2A}$

- In ductile material failure in tension is initiated by shear stress i.e. the failure occurs across the shear planes at  $45^\circ$  (where it is maximum) to the applied load.

**Let us clear a concept about a common mistake:** The angle  $\theta$  is not between the applied load and the plane. It is between the planes XX and YY. But if in any question the angle between the applied load and the plane is given don't take it as  $\theta$ . The angle between the applied load and the plane is  $90^\circ - \theta$ . In this case you have to use the above formula as  $\sigma_n = \frac{P}{A} \cos^2(90^\circ - \theta)$  and  $\tau = \frac{P}{2A} \sin(180^\circ - 2\theta)$  where  $\theta$  is the angle between the applied load and the plane. Carefully observe the following two figures it will be clear.



**Let us take an example:** A metal block of  $100 \text{ mm}^2$  cross sectional area carries an axial tensile load of 10 kN. For a plane inclined at  $30^\circ$  with the direction of applied load, calculate:

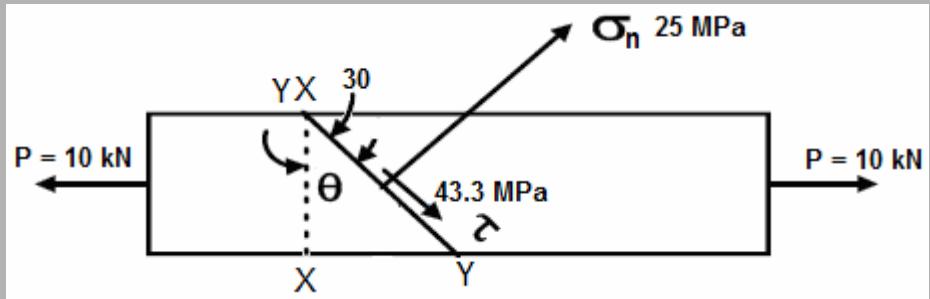
- Normal stress
- Shear stress
- Maximum shear stress.

**Answer:** Here  $\theta = 90^\circ - 30^\circ = 60^\circ$

$$(a) \text{ Normal stress } (\sigma_n) = \frac{P}{A} \cos^2 \theta = \frac{10 \times 10^3 \text{ N}}{100 \text{ mm}^2} \times \cos^2 60^\circ = 25 \text{ MPa}$$

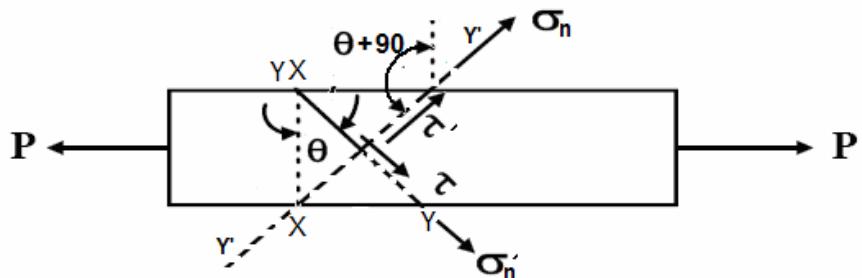
$$(b) \text{ Shear stress } (\tau) = \frac{P}{2A} \sin 2\theta = \frac{10 \times 10^3 \text{ N}}{2 \times 100 \text{ mm}^2} \times \sin 120^\circ = 43.3 \text{ MPa}$$

$$(c) \text{ Maximum shear stress } (\tau_{\max}) = \frac{P}{2A} = \frac{10 \times 10^3 \text{ N}}{2 \times 100 \text{ mm}^2} = 50 \text{ MPa}$$



- **Complementary stresses**

Now if we consider the stresses on an oblique plane YY' which is perpendicular to the previous plane YY. The stresses on this plane are known as complementary stresses. **Complementary normal stress is  $\sigma'_n$**  and **complementary shear stress is  $\tau'$** . The following figure shows all the four stresses. To obtain the stresses  $\sigma'_n$  and  $\tau'$  we need only to replace  $\theta$  by  $\theta + 90^\circ$  in the previous equation. The angle  $\theta + 90^\circ$  is known as **aspect angle**.



Therefore

$$\sigma'_n = \frac{P}{A} \cos^2 (90^\circ + \theta) = \frac{P}{A} \sin^2 \theta$$

$$\tau' = \frac{P}{2A} \sin 2(90^\circ + \theta) = -\frac{P}{2A} \sin 2\theta$$

It is clear  $\sigma'_n + \sigma_n = \frac{P}{A}$  and  $\tau' = -\tau$

i.e. **Complementary shear stresses are always equal in magnitude but opposite in sign.**

- **Sign of Shear stress**

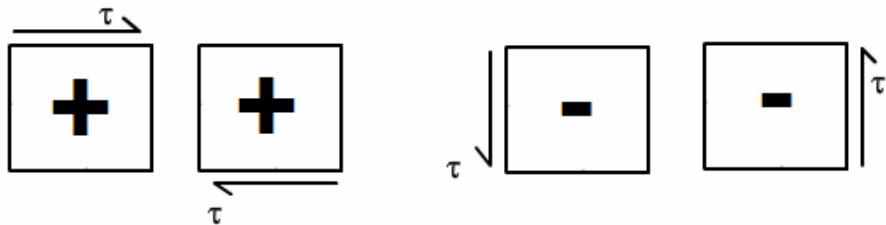
For sign of shear stress following rule have to be followed:

## Chapter-2

## Principal Stress and Strain

The shear stress  $\tau$  on any face of the element will be considered **positive** when it has a **clockwise** moment with respect to a centre inside the element. If the moment is **counter-clockwise** with respect to a centre inside the element, the shear stress is **negative**.

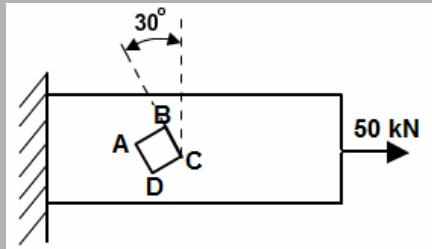
**Note:** The convention is opposite to that of moment of force. Shear stress tending to turn clockwise is positive and tending to turn counter clockwise is negative.



**Let us take an example:** A prismatic bar of  $500 \text{ mm}^2$  cross sectional area is axially loaded with a tensile force of  $50 \text{ kN}$ . Determine all the stresses acting on an element which makes  $30^\circ$  inclination with the vertical plane.

**Answer:** Take an small element ABCD in  $30^\circ$  plane as shown in figure below,

Given, Area of cross-section,  $A = 500 \text{ mm}^2$ , Tensile force ( $P$ ) =  $50 \text{ kN}$



$$\text{Normal stress on } 30^\circ \text{ inclined plane, } (\sigma_n) = \frac{P}{A} \cos^2 \theta = \frac{50 \times 10^3 \text{ N}}{500 \text{ mm}^2} \times \cos^2 30^\circ = 75 \text{ MPa} \text{ (+ive means tensile).}$$

$$\text{Shear stress on } 30^\circ \text{ planes, } (\tau) = \frac{P}{2A} \sin 2\theta = \frac{50 \times 10^3 \text{ N}}{2 \times 500 \text{ mm}^2} \times \sin(2 \times 30^\circ) = 43.3 \text{ MPa}$$

(+ive means clockwise)

$$\text{Complementary stress on } (\theta) = 90 + 30 = 120^\circ$$

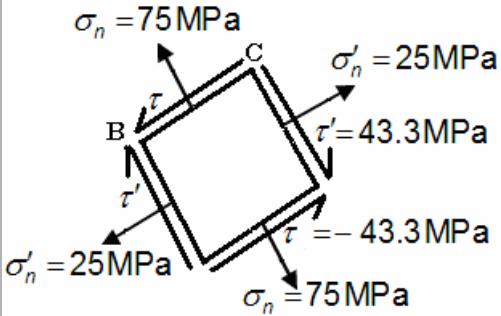
$$\text{Normal stress on } 120^\circ \text{ inclined plane, } (\sigma'_n) = \frac{P}{A} \cos^2 \theta = \frac{50 \times 10^3 \text{ N}}{500 \text{ mm}^2} \times \cos^2 120^\circ = 25 \text{ MPa}$$

(+ive means tensile)

$$\text{Shear stress on } 120^\circ \text{ inclined plane, } (\tau') = \frac{P}{2A} \sin 2\theta = \frac{50 \times 10^3 \text{ N}}{2 \times 500 \text{ mm}^2} \times \sin(2 \times 120^\circ) = -43.3 \text{ MPa}$$

(-ive means counter clockwise)

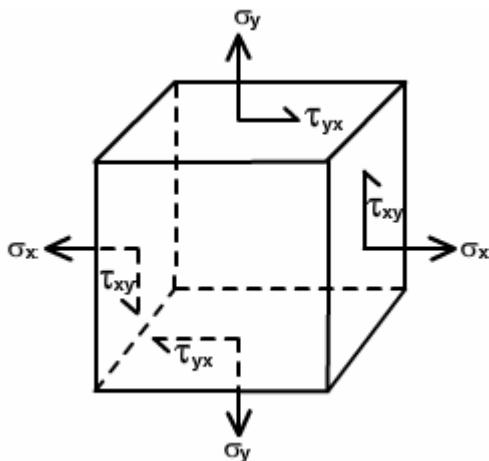
State of stress on the element ABCD is given below (magnifying)



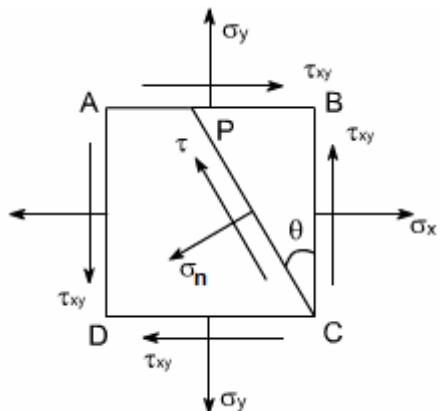
## 2.3 Complex Stresses (2-D Stress system)

i.e. Material subjected to combined direct and shear stress

We now consider a complex stress system below. The given figure ABCD shows on small element of material



Stresses in three dimensional element



Stresses in cross-section of the element

$\sigma_x$  and  $\sigma_y$  are normal stresses and may be tensile or compressive. We know that normal stress may come from direct force or bending moment.  $\tau_{xy}$  is shear stress. We know that shear stress may come from direct shear force or torsion and  $\tau_{xy}$  and  $\tau_{yx}$  are complementary and

$$\tau_{xy} = \tau_{yx}$$

Let  $\sigma_n$  is the normal stress and  $\tau$  is the shear stress on a plane at angle  $\theta$ .

Considering the equilibrium of the element we can easily get

$$\text{Normal stress}(\sigma_n) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

and

$$\text{Shear stress}(\tau) = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

Above two equations are coming from considering equilibrium. They do not depend on material properties and are valid for elastic and inelastic behavior.

- Location of planes of maximum stress

**(a) Normal stress,  $(\sigma_n)_{\max}$**

For  $\sigma_n$  maximum or minimum

$$\frac{\partial \sigma_n}{\partial \theta} = 0, \text{ where } \sigma_n = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\text{or } -\frac{(\sigma_x - \sigma_y)}{2} \times (\sin 2\theta) \times 2 + \tau_{xy} (\cos 2\theta) \times 2 = 0 \quad \text{or } \tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

**(b) Shear stress,  $\tau_{\max}$**

For  $\tau$  maximum or minimum

$$\frac{\partial \tau}{\partial \theta} = 0, \text{ where } \tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$\text{or } \frac{\sigma_x - \sigma_y}{2} (\cos 2\theta) \times 2 - \tau_{xy} (-\sin 2\theta) \times 2 = 0$$

$$\text{or } \cot 2\theta = \frac{-\tau_{xy}}{\sigma_x - \sigma_y}$$

**Let us take an example:** At a point in a crank shaft the stresses on two mutually perpendicular planes are 30 MPa (tensile) and 15 MPa (tensile). The shear stress across these planes is 10 MPa. Find the normal and shear stress on a plane making an angle  $30^\circ$  with the plane of first stress. Find also magnitude and direction of resultant stress on the plane.

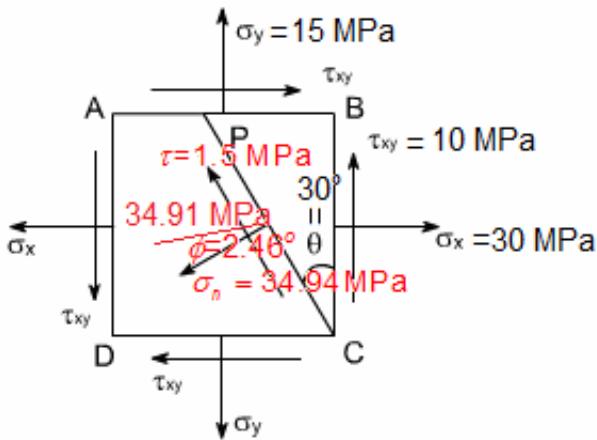
Answer: Given  $\sigma_x = +25 \text{ MPa (tensile)}$ ,  $\sigma_y = +15 \text{ MPa (tensile)}$ ,  $\tau_{xy} = 10 \text{ MPa}$  and  $40^\circ$

$$\begin{aligned} \text{Therefore, Normal stress } (\sigma_n) &= \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \\ &= \frac{30+15}{2} + \frac{30-15}{2} \cos(2 \times 30^\circ) + 10 \sin(2 \times 30^\circ) = 34.91 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \text{Shear stress } (\tau) &= \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta \\ &= \frac{30-15}{2} \sin(2 \times 30^\circ) - 10 \cos(2 \times 30^\circ) = 1.5 \text{ MPa} \end{aligned}$$

$$\text{Resultant stress } (\sigma_r) = \sqrt{(34.91)^2 + 1.5^2} = 34.94 \text{ MPa}$$

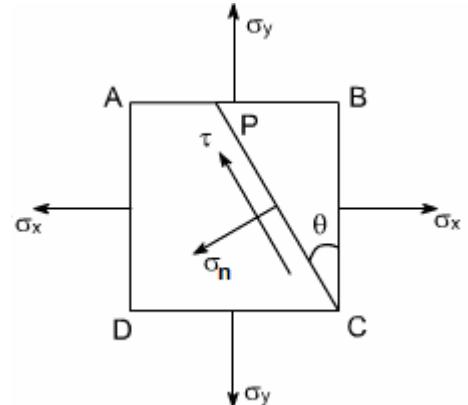
$$\text{and Obliquity } (\phi), \tan \phi = \frac{\tau}{\sigma_n} = \frac{1.5}{34.91} \Rightarrow \phi = 2.46^\circ$$



## 2.4 Bi-axial stress

Let us now consider a stressed element ABCD where  $\tau_{xy}=0$ , i.e. only  $\sigma_x$  and  $\sigma_y$  is there. This type of stress is known as bi-axial stress. In the previous equation if you put  $\tau_{xy}=0$  we get Normal stress,  $\sigma_n$  and shear stress,  $\tau$  on a plane at angle  $\theta$ .

- Normal stress,  $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$
- Shear/Tangential stress,  $\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$
- For complementary stress, aspect angle =  $\theta + 90^\circ$
- Aspect angle 'θ' varies from 0 to  $\pi/2$
- Normal stress  $\sigma_n$  varies between the values  $\sigma_x(\theta = 0)$  &  $\sigma_y(\theta = \pi/2)$



**Let us take an example:** The principal tensile stresses at a point across two perpendicular planes are 100 MPa and 50 MPa. Find the normal and tangential stresses and the resultant stress and its obliquity on a plane at  $20^\circ$  with the major principal plane

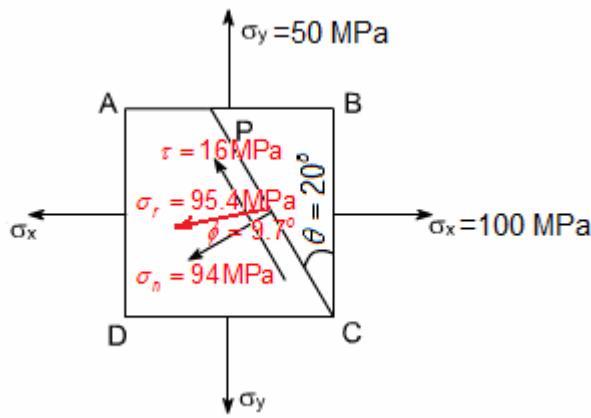
**Answer:** Given  $\sigma_x = 100 \text{ MPa}$  (tensile),  $\sigma_y = 50 \text{ MPa}$  (tensile) and  $\theta = 20^\circ$

$$\text{Normal stress, } (\sigma_n) = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta = \frac{100 + 50}{2} + \frac{100 - 50}{2} \cos(2 \times 20^\circ) = 94 \text{ MPa}$$

$$\text{Shear stress, } (\tau) = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta = \frac{100 - 50}{2} \sin(2 \times 20^\circ) = 16 \text{ MPa}$$

$$\text{Resultant stress } (\sigma_r) = \sqrt{94^2 + 16^2} = 95.4 \text{ MPa}$$

$$\text{Therefore angle of obliquity, } (\phi) = \tan^{-1} \left( \frac{\tau}{\sigma_n} \right) = \tan^{-1} \left( \frac{16}{94} \right) = 9.7^\circ$$



- We may derive uni-axial stress on oblique plane from

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

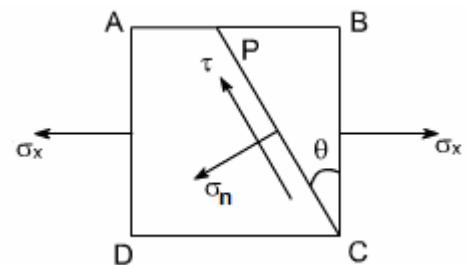
and  $\tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$

Just put  $\sigma_y = 0$  and  $\tau_{xy} = 0$

Therefore,

$$\sigma_n = \frac{\sigma_x + 0}{2} + \frac{\sigma_x - 0}{2} \cos 2\theta = \frac{1}{2} \sigma_x (1 + \cos 2\theta) = \sigma_x \cos^2 \theta$$

and  $\tau = \frac{\sigma_x - 0}{2} \sin 2\theta = \frac{\sigma_x}{2} \sin 2\theta$



## 2.5 Pure Shear

- Pure shear is a particular case of bi-axial stress where

$$\sigma_x = -\sigma_y$$

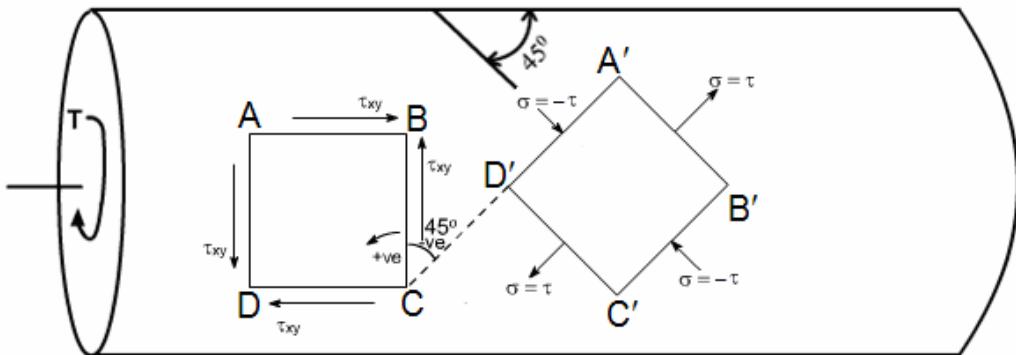
Note:  $\sigma_x$  or  $\sigma_y$  which one is compressive that is immaterial but one should be tensile and other should be compressive and equal magnitude. If  $\sigma_x = 100 \text{ MPa}$  then  $\sigma_y$  must be  $-100 \text{ MPa}$  otherwise if  $\sigma_y = 100 \text{ MPa}$  then  $\sigma_x$  must be  $-100 \text{ MPa}$ .

- In case of pure shear on  $45^\circ$  planes

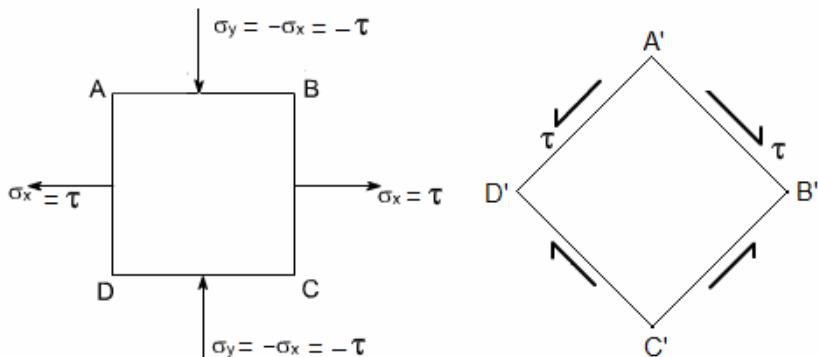
$$\tau_{\max} = \pm \sigma_x ; \sigma_n = 0 \text{ and } \sigma'_n = 0$$

- We may depict the pure shear in an element by following two ways

(a) In a torsion member, as shown below, an element ABCD is in pure shear (only shear stress is present in this element) in this member at  $45^\circ$  plane an element  $A'B'C'D'$  is also in pure shear where  $\sigma_x = -\sigma_y$  but in this element no shear stress is there.



(b) In a bi-axial state of stress a member, as shown below, an element ABCD in pure shear where  $\sigma_x = -\sigma_y$  but in this element no shear stress is there and an element  $A'B'C'D'$  at  $45^\circ$  plane is also in pure shear (only shear stress is present in this element).



**Let us take an example:** See the in the Conventional question answer section in this chapter and the question is “*Conventional Question IES-2007*”

## 2.6 Stress Tensor

- **State of stress at a point ( 3-D)**

Stress acts on every surface that passes through the point. We can use three mutually perpendicular planes to describe the stress state at the point, which we approximate as a cube each of the three planes has one normal component & two shear components therefore, 9 components necessary to define stress at a point 3 normal and 6 shear stress.

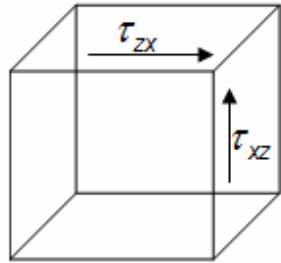
Therefore, ***we need nine components, to define the state of stress at a point***

$$\begin{array}{lll} \sigma_x & \tau_{xy} & \tau_{xz} \\ \sigma_y & \tau_{yx} & \tau_{yz} \\ \sigma_z & \tau_{zx} & \tau_{zy} \end{array}$$

*For cube to be in equilibrium (at rest: not moving, not spinning)*

## Chapter-2

## Principal Stress and Strain



$$\begin{aligned}\tau_{xy} &= \tau_{yx} \\ \tau_{xz} &= \tau_{zx} \\ \tau_{yz} &= \tau_{zy}\end{aligned}$$

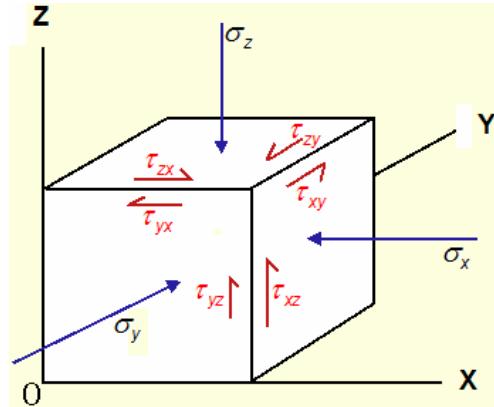
If they don't offset, block spins therefore,  
only six are independent.

The nine components (six of which are independent) can be written in matrix form

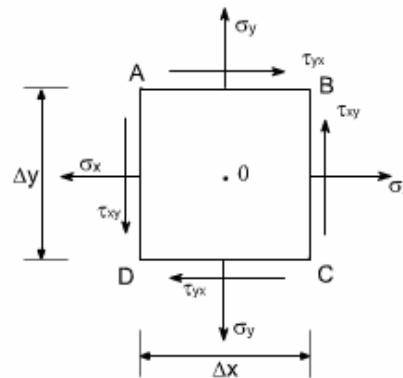
$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \text{ or } \tau_{ij} = \begin{pmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{pmatrix} = \begin{pmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{pmatrix} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

This is the stress tensor

Components on diagonal are normal stresses; off are shear stresses



### • State of stress at an element (2-D)



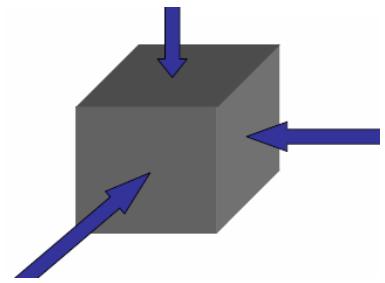
## 2.7 Principal stress and Principal plane

- When examining stress at a point, it is possible to choose **three mutually perpendicular planes** on which **no shear** stresses exist in three dimensions, one combination of orientations for the three mutually perpendicular planes will cause the shear stresses on all three planes to go to zero *this is the state defined by the principal stresses.*

## Chapter-2

## Principal Stress and Strain

- Principal stresses are normal stresses that are orthogonal to each other
- Principal planes are the planes across which principal stresses act (faces of the cube) for principal stresses (shear stresses are zero)
- Major Principal Stress



$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- Minor principal stress

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

- Position of principal planes

$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

- Maximum shear stress

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

**Let us take an example:** In the wall of a cylinder the state of stress is given by,  $\sigma_x = 85\text{ MPa}$  (compressive),  $\sigma_y = 25\text{ MPa}$  (tensile) and shear stress ( $\tau_{xy}$ ) =  $60\text{ MPa}$

Calculate the principal planes on which they act. Show it in a figure.

**Answer:** Given  $\sigma_x = -85\text{ MPa}$ ,  $\sigma_y = 25\text{ MPa}$ ,  $\tau_{xy} = 60\text{ MPa}$

## Chapter-2

## Principal Stress and Strain

$$\text{Major principal stress } (\sigma_1) = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-85 + 25}{2} + \sqrt{\left(\frac{-85 - 25}{2}\right)^2 + 60^2} = 51.4 \text{ MPa}$$

$$\text{Minor principal stress } (\sigma_2) = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{-85 + 25}{2} - \sqrt{\left(\frac{-85 - 25}{2}\right)^2 + 60^2}$$

$$= -111.4 \text{ MPa} \text{ i.e. } 111.4 \text{ MPa (Compressive)}$$

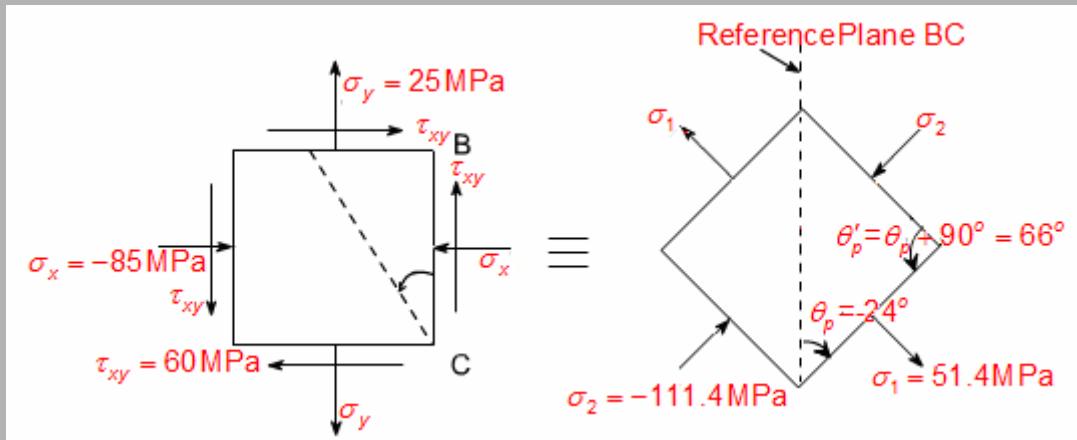
For principal planes

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 60}{-85 - 25}$$

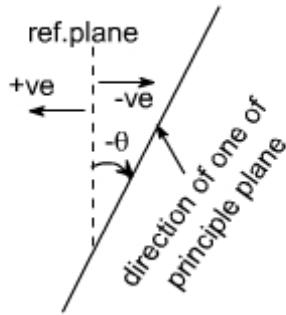
or  $\theta_p = -24^\circ$  it is for  $\sigma_1$

Complementary plane  $\theta'_p = \theta_p + 90^\circ = 66^\circ$  it is for  $\sigma_2$

The Figure showing state of stress and principal stresses is given below



The direction of one principle plane and the principle stresses acting on this would be  $\sigma_1$  when is acting normal to this plane, now the direction of other principal plane would be  $90^\circ + \theta_p$  because the principal planes are the two mutually perpendicular plane, hence rotate the another plane  $90^\circ + \theta_p$  in the same direction to get the another plane, now complete the material element as  $\theta_p$  is negative that means we are measuring the angles in the opposite direction to the reference plane BC. The following figure gives clear idea about negative and positive  $\theta_p$ .



## 2.8 Mohr's circle for plane stress

- The transformation equations of plane stress can be represented in a graphical form which is popularly known as **Mohr's circle**.
- Though the transformation equations are sufficient to get the normal and shear stresses on any plane at a point, with Mohr's circle one can easily visualize their variation with respect to plane orientation  $\theta$ .
- Equation of Mohr's circle**

$$\text{We know that normal stress, } \sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\text{And Tangential stress, } \tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta$$

$$\text{Rearranging we get, } \left( \sigma_n - \frac{\sigma_x + \sigma_y}{2} \right) = \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta \dots \dots \dots \text{(i)}$$

$$\text{and } \tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta \dots \dots \dots \text{(ii)}$$

A little consideration will show that the above two equations are the equations of a circle with  $\sigma_n$  and  $\tau$  as its coordinates and  $2\theta$  as its parameter.

If the parameter  $2\theta$  is eliminated from the equations, (i) & (ii) then the significance of them will become clear.

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \text{ and } R = \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$\text{Or } \left( \sigma_n - \sigma_{avg} \right)^2 + \tau_{xy}^2 = R^2$$

It is the equation of a circle with **centre**,  $(\sigma_{avg}, 0)$  i.e.  $\left( \frac{\sigma_x + \sigma_y}{2}, 0 \right)$

and radius,  $R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

- **Construction of Mohr's circle**

**Convention for drawing**

- A  $\tau_{xy}$  that is clockwise (positive) on a face resides above the  $\sigma$  axis; a  $\tau_{xy}$  anticlockwise (negative) on a face resides below  $\sigma$  axis.
- Tensile stress will be positive and plotted right of the origin O. Compressive stress will be negative and will be plotted left to the origin O.
- An angle  $\theta$  on real plane transfers as an angle  $2\theta$  on Mohr's circle plane.

We now construct Mohr's circle in the following stress conditions

I. Bi-axial stress when  $\sigma_x$  and  $\sigma_y$  known and  $\tau_{xy} = 0$

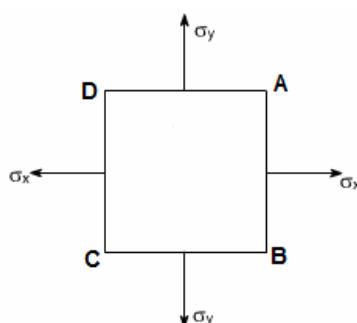
II. Complex state of stress ( $\sigma_x$ ,  $\sigma_y$  and  $\tau_{xy}$  known)

**I. Constant of Mohr's circle for Bi-axial stress (when only  $\sigma_x$  and  $\sigma_y$  known)**

If  $\sigma_x$  and  $\sigma_y$  both are tensile or both compressive sign of  $\sigma_x$  and  $\sigma_y$  will be same and this state of stress is known as "like stresses" if one is tensile and other is compressive sign of  $\sigma_x$  and  $\sigma_y$  will be opposite and this state of stress is known as 'unlike stress'.

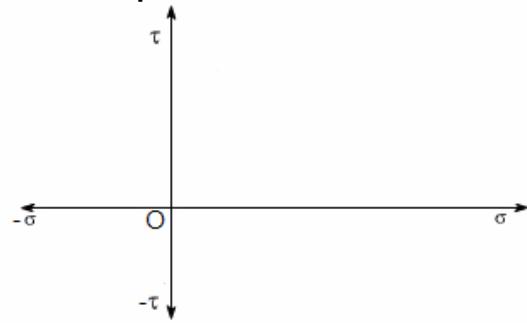
**• Construction of Mohr's circle for like stresses (when  $\sigma_x$  and  $\sigma_y$  are same type of stress)**

**Step-I:** Label the element ABCD and draw all stresses.

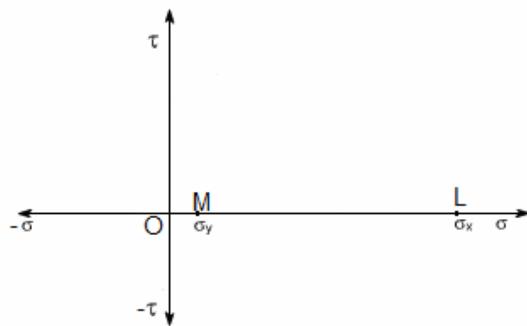


**Step-II:** Set up axes for the direct stress (as abscissa) i.e., in x-axis and shear stress (as ordinate) i.e. in Y-axis

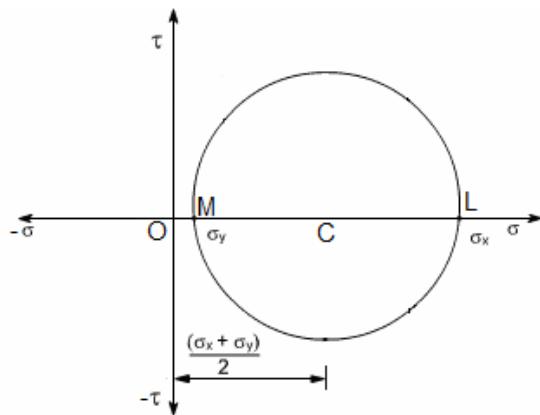
## Principal Stress and Strain



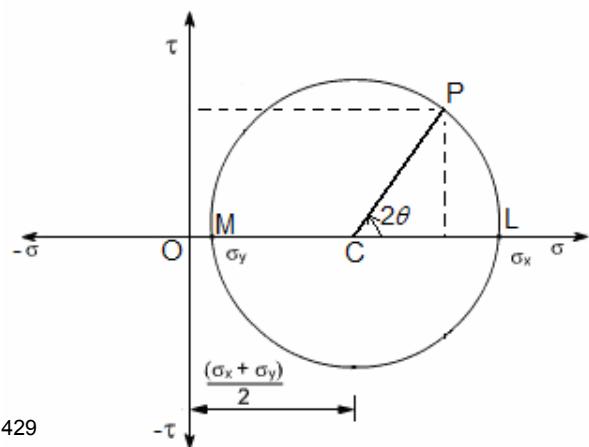
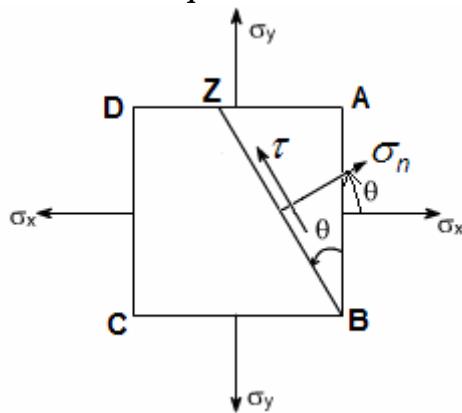
**Step-III:** Using sign convention and some suitable scale, plot the stresses on two adjacent faces e.g. AB and BC on the graph. Let OL and OM equal to  $\sigma_x$  and  $\sigma_y$  respectively on the axis O $\sigma$ .



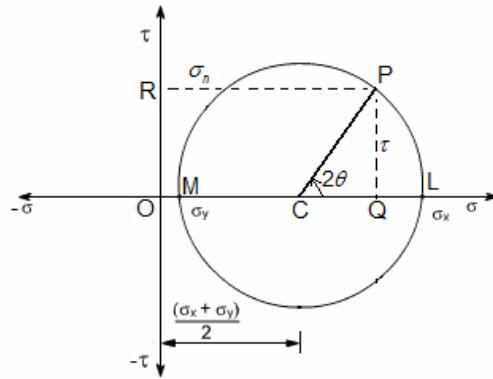
**Step-IV:** Bisect ML at C. With C as centre and CL or CM as radius, draw a circle. It is the Mohr's circle.



**Step-V:** At the centre C draw a line CP at an angle  $2\theta$ , in the same direction as the normal to the plane makes with the direction of  $\sigma_x$ . **The point P represents the state of stress at plane ZB.**



**Step-VI: Calculation,** Draw a perpendicular PQ and PR where  $PQ = \tau$  and  $PR = \sigma_n$



$$OC = \frac{\sigma_x + \sigma_y}{2} \text{ and } MC = CL = CP = \frac{\sigma_x - \sigma_y}{2}$$

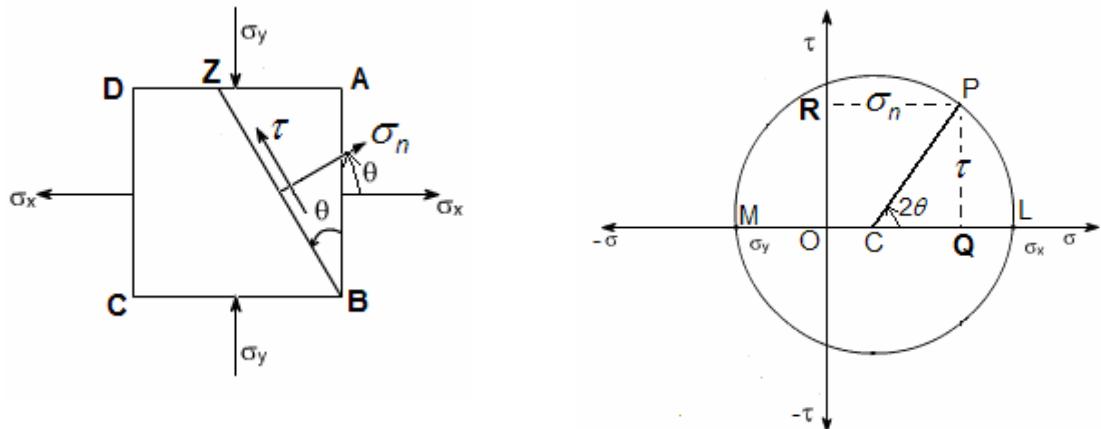
$$PR = \sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$PQ = \tau = CP \sin 2\theta = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta$$

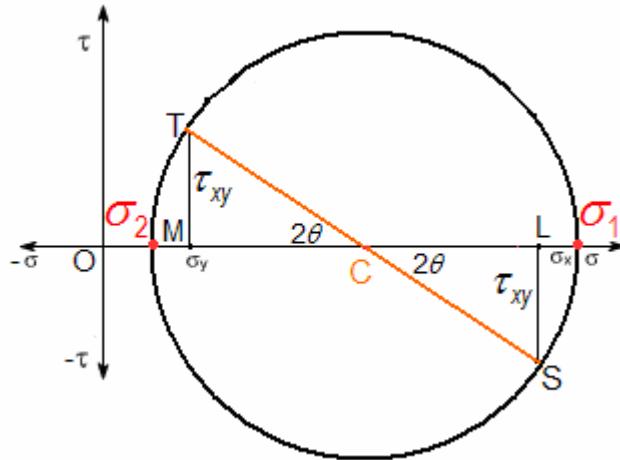
[Note: In the examination you only draw final figure (which is in Step-V) and follow the procedure step by step so that no mistakes occur.]

- **Construction of Mohr's circle for unlike stresses (when  $\sigma_x$  and  $\sigma_y$  are opposite in sign)**

Follow the same steps which we followed for construction for 'like stresses' and finally will get the figure shown below.

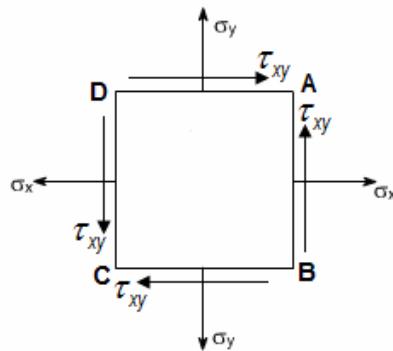


**Note:** For construction of Mohr's circle for principal stresses when ( $\sigma_1$  and  $\sigma_2$  is known) then follow the same steps of Constant of Mohr's circle for Bi-axial stress (when only  $\sigma_x$  and  $\sigma_y$  known) just change the  $\sigma_x = \sigma_1$  and  $\sigma_y = \sigma_2$

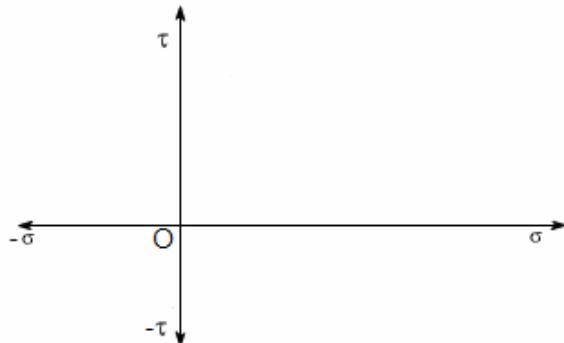


**II. Construction of Mohr's circle for complex state of stress ( $\sigma_x$ ,  $\sigma_y$  and  $\tau_{xy}$  known)**

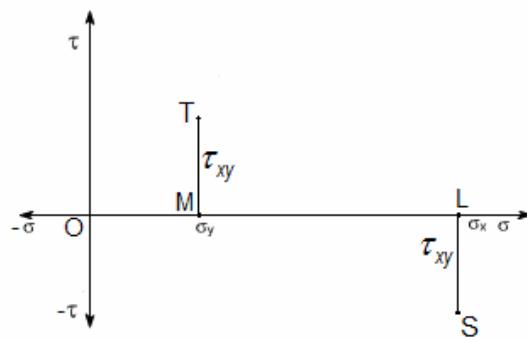
**Step-I:** Label the element ABCD and draw all stresses.



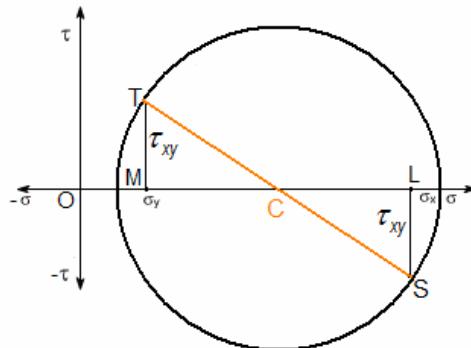
**Step-II:** Set up axes for the direct stress (as abscissa) i.e., in x-axis and shear stress (as ordinate) i.e. in Y-axis



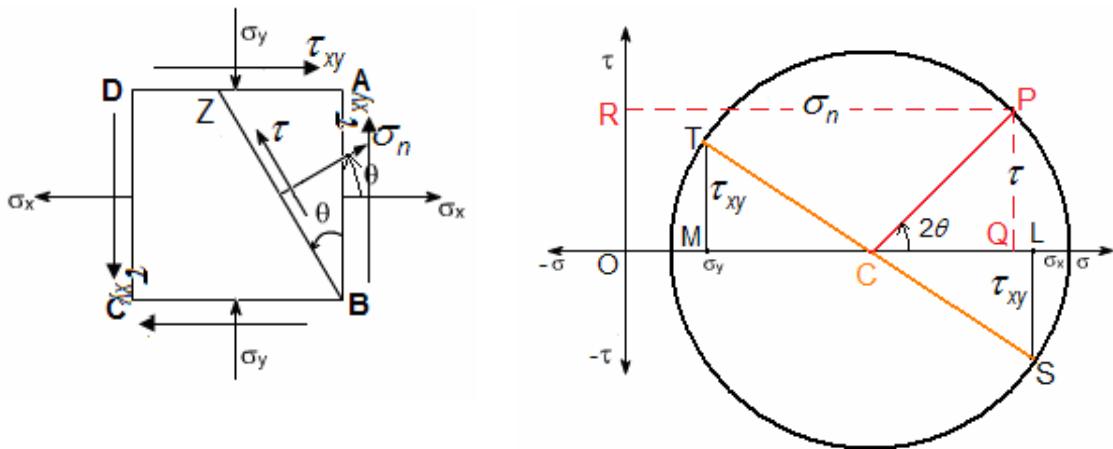
**Step-III:** Using sign convention and some suitable scale, plot the stresses on two adjacent faces e.g. AB and BC on the graph. Let OL and OM equal to  $\sigma_x$  and  $\sigma_y$  respectively on the axis  $O\sigma$ . Draw LS perpendicular to  $O\sigma$  axis and equal to  $\tau_{xy}$  i.e.  $LS = \tau_{xy}$ . Here LS is downward as  $\tau_{xy}$  on AB face is (-ive) and draw MT perpendicular to  $O\sigma$  axis and equal to  $\tau_{xy}$  i.e.  $MT = \tau_{xy}$ . Here MT is upward as  $\tau_{xy}$  BC face is (+ive).



**Step-IV:** Join ST and it will cut  $\sigma\sigma$  axis at C. With C as centre and CS or CT as radius, draw circle. It is the Mohr's circle.



**Step-V:** At the centre draw a line CP at an angle  $2\theta$  in the same direction as the normal to the plane makes with the direction of  $\sigma_x$ .



**Step-VI: Calculation,** Draw a perpendicular PQ and PR where  $PQ = \tau$  and  $PR = \sigma_n$

$$\text{Centre, } OC = \frac{\sigma_x + \sigma_y}{2}$$

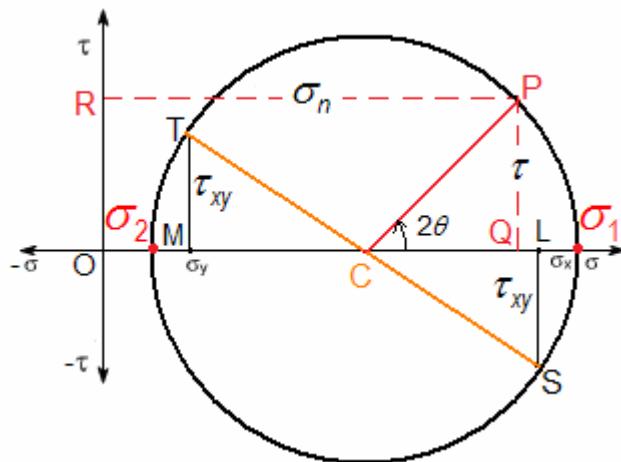
$$\text{Radius } CS = \sqrt{(CL)^2 + (LS)^2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = CT = CP$$

$$PR = \sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$PQ = \tau = \frac{\sigma_x - \sigma_y}{2} \sin 2\theta - \tau_{xy} \cos 2\theta.$$

[Note: In the examination you only draw final figure (which is in Step-V) and follow the procedure step by step so that no mistakes occur.]

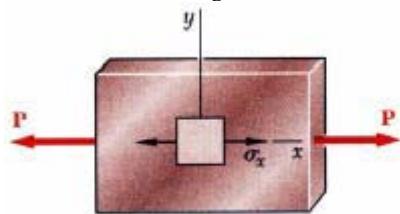
**Note:** The intersections of  $O\sigma$  axis are two principal stresses, as shown below.



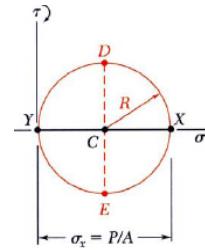
**Let us take an example:** See the in the Conventional question answer section in this chapter and the question is "Conventional Question IES-2000"

## 2.9 Mohr's circle for some special cases:

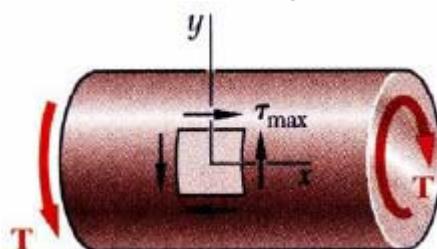
i) Mohr's circle for axial loading:



$$\sigma_x = \frac{P}{A}; \quad \sigma_y = \tau_{xy} = 0$$

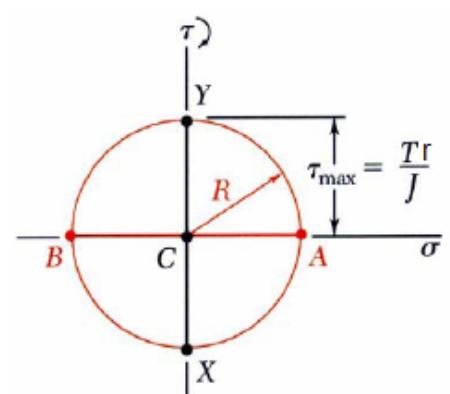


ii) Mohr's circle for torsional loading:

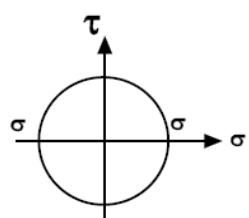
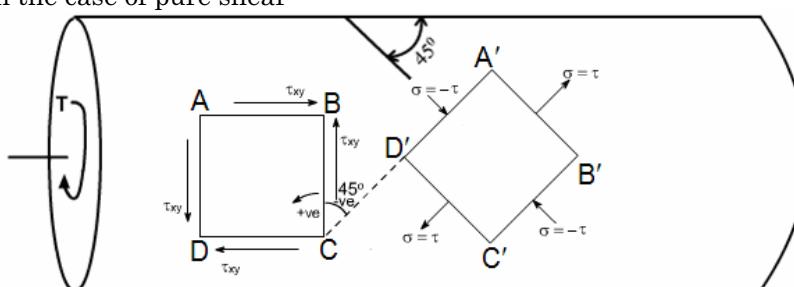


$$\tau_{xy} = \frac{Tr}{J}; \quad \sigma_x = \sigma_y = 0$$

It is a case of pure shear



iii) In the case of pure shear

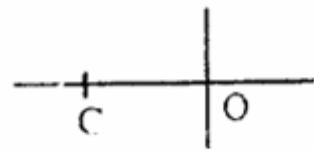
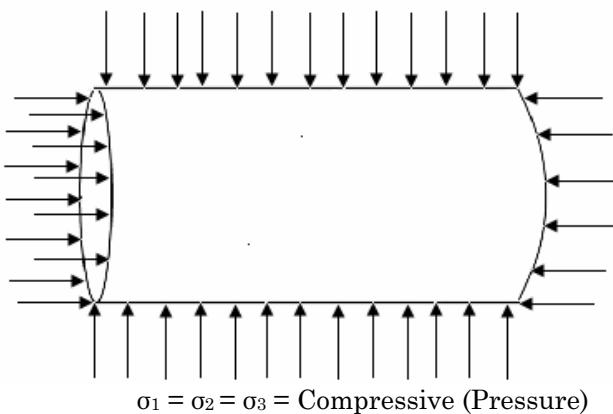


$(\sigma_1 = -\sigma_2 \text{ and } \sigma_3 = 0)$  Page 73 of 429

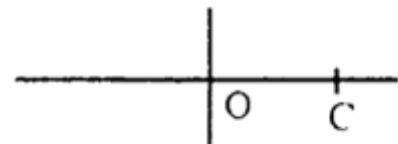
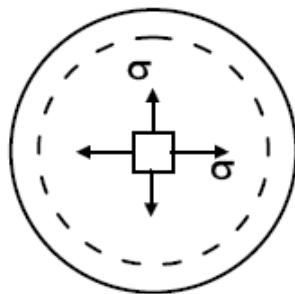
$$\sigma_x = -\sigma_y$$

$$\tau_{\max} = \pm \sigma_x$$

iv) A shaft compressed all round by a hub

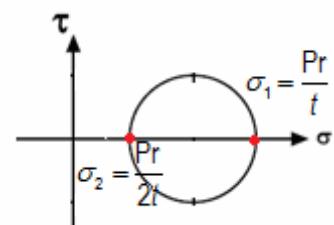
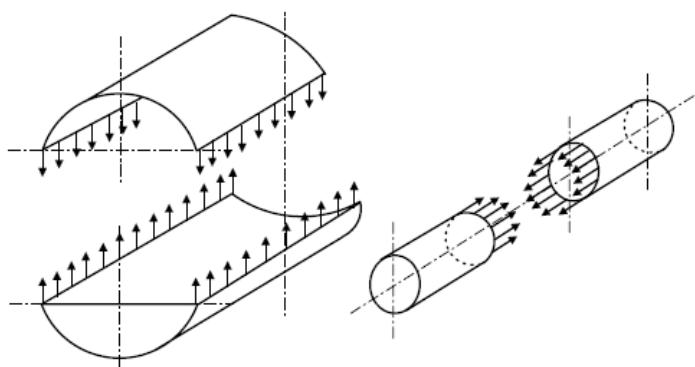


v) Thin spherical shell under internal pressure



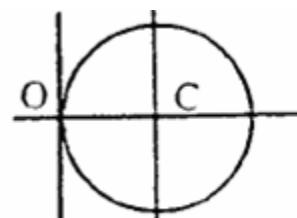
$$\sigma_1 = \sigma_2 = \frac{pr}{2t} = \frac{pD}{4t} \text{ (tensile)}$$

vi) Thin cylinder under pressure



$$\sigma_1 = \frac{pD}{2t} = \frac{pr}{t} \text{ (tensile)} \text{ and } \sigma_2 = \frac{pd}{4t} = \frac{pr}{2t} \text{ (tensile)}$$

vii) Bending moment applied at the free end of a cantilever



$$\text{Only bending stress, } \sigma_1 = \frac{My}{I} \text{ and } \sigma_2 = \tau_{xy} = 0$$

### Normal strain

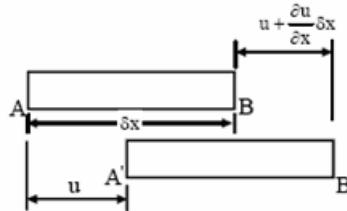
Let us consider an element AB of infinitesimal length  $\delta x$ . After deformation of the actual body if displacement of end A is  $u$ , that of end B is  $u + \frac{\partial u}{\partial x} \cdot \delta x$ . This gives an increase in length of element AB

$$\text{is } \left( u + \frac{\partial u}{\partial x} \cdot \delta x - u \right) = \frac{\partial u}{\partial x} \delta x \text{ and therefore the strain in } x\text{-direction is } \varepsilon_x = \frac{\partial u}{\partial x}$$

$$\text{Similarly, strains in } y \text{ and } z \text{ directions are } \varepsilon_y = \frac{\partial v}{\partial x} \text{ and } \varepsilon_z = \frac{\partial w}{\partial z}.$$

Therefore, we may write the three normal strain components

$$\varepsilon_x = \frac{\partial u}{\partial x}; \quad \varepsilon_y = \frac{\partial v}{\partial y}; \quad \text{and} \quad \varepsilon_z = \frac{\partial w}{\partial z}.$$



Change in length of an infinitesimal element.

### Shear strain

Let us consider an element ABCD in x-y plane and let the displaced position of the element be A'B'C'D'. This gives shear strain in x-y plane as  $\gamma_{xy} = \alpha + \beta$  where  $\alpha$  is the angle made by the displaced line B'C' with the vertical and  $\beta$  is the angle made by the displaced line A'D' with the horizontal. This gives  $\alpha = \frac{\partial u}{\partial y} = \frac{\partial u}{\partial y}$  and  $\beta = \frac{\partial v}{\partial x} = \frac{\partial v}{\partial x}$

We may therefore write the three shear strain components as

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}; \quad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \quad \text{and} \quad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

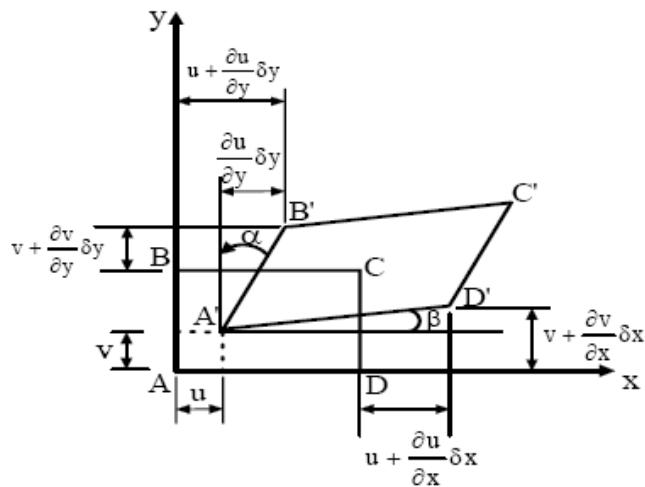
Therefore the state of strain at a point can be completely described by the **six strain components** and the strain components in their turns can be completely defined by the displacement components  $u, v$ , and  $w$ .

Therefore, the complete strain matrix can be written as

## Chapter-2

## Principal Stress and Strain

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ 0 & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$



Shear strain associated with the distortion of an infinitesimal element.

## Strain Tensor

The three normal strain components are

$$\varepsilon_x = \varepsilon_{xx} = \frac{\partial u}{\partial x}; \quad \varepsilon_y = \varepsilon_{yy} = \frac{\partial v}{\partial y} \quad \text{and} \quad \varepsilon_z = \varepsilon_{zz} = \frac{\partial w}{\partial z}.$$

The three shear strain components are

$$\varepsilon_{xy} = \frac{\gamma_{xy}}{2} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right); \quad \varepsilon_{yz} = \frac{\gamma_{yz}}{2} = \frac{1}{2} \left( \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \right) \quad \text{and} \quad \varepsilon_{zx} = \frac{\gamma_{zx}}{2} = \frac{1}{2} \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$

Therefore the strain tensor is

$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} = \begin{bmatrix} \varepsilon_{xx} & \frac{\gamma_{xy}}{2} & \frac{\gamma_{xz}}{2} \\ \frac{\gamma_{yx}}{2} & \varepsilon_{yy} & \frac{\gamma_{yz}}{2} \\ \frac{\gamma_{zx}}{2} & \frac{\gamma_{zy}}{2} & \varepsilon_{zz} \end{bmatrix}$$

## Constitutive Equation

The constitutive equations relate stresses and strains and in linear elasticity. We know from the Hook's law  $(\sigma) = E \cdot \varepsilon$

## Chapter-2

## Principal Stress and Strain

It is known that  $\sigma_x$  produces a strain of  $\frac{\sigma_x}{E}$  in x-direction

and Poisson's effect gives  $-\mu \frac{\sigma_x}{E}$  in y-direction and  $-\mu \frac{\sigma_x}{E}$  in z-direction.

Therefore we may write the generalized Hook's law as

$$\epsilon_x = \frac{1}{E} [\sigma_x - \mu(\sigma_y + \sigma_z)], \quad \epsilon_y = \frac{1}{E} [\sigma_y - \mu(\sigma_z + \sigma_x)] \quad \text{and} \quad \epsilon_z = \frac{1}{E} [\sigma_z - \mu(\sigma_x + \sigma_y)]$$

It is also known that the shear stress,  $\tau = G\gamma$ , where G is the shear modulus and  $\gamma$  is shear strain.

We may thus write the three strain components as

$$\gamma_{xy} = \frac{\tau_{xy}}{G}, \quad \gamma_{yz} = \frac{\tau_{yz}}{G} \quad \text{and} \quad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

In general each strain is dependent on each stress and we may write

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} & K_{13} & K_{14} & K_{15} & K_{16} \\ K_{21} & K_{22} & K_{23} & K_{24} & K_{25} & K_{26} \\ K_{31} & K_{32} & K_{33} & K_{34} & K_{35} & K_{36} \\ K_{41} & K_{42} & K_{43} & K_{44} & K_{45} & K_{46} \\ K_{51} & K_{52} & K_{53} & K_{54} & K_{55} & K_{56} \\ K_{61} & K_{62} & K_{63} & K_{64} & K_{65} & K_{66} \end{bmatrix} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{bmatrix}$$

∴ The number of elastic constant is 36 (For anisotropic materials)

For isotropic material

$$K_{11} = K_{22} = K_{33} = \frac{1}{E}$$

$$K_{44} = K_{55} = K_{66} = \frac{1}{G}$$

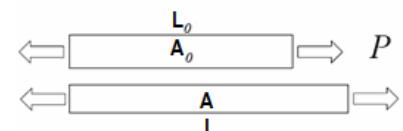
$$K_{12} = K_{13} = K_{21} = K_{23} = K_{31} = K_{32} = -\frac{\mu}{E}$$

Rest of all elements in K matrix are zero.

For isotropic material only two independent elastic constant is there say E and G.

### • 1-D Strain

Let us take an example: A rod of cross sectional area  $A_o$  is loaded by a tensile force P.



It's stresses  $\sigma_x = \frac{P}{A_o}$ ,  $\sigma_y = 0$ , and  $\sigma_z = 0$

### 1-D state of stress or Uni-axial state of stress

$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ or } \tau_{ij} = \begin{pmatrix} \tau_{xx} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} \sigma_x & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Therefore strain components are

$$\epsilon_x = \frac{\sigma_x}{E}; \epsilon_y = -\mu \frac{\sigma_x}{E} = -\mu \epsilon_x; \text{ and } \epsilon_z = -\mu \frac{\sigma_x}{E} = -\mu \epsilon_x$$

**1-D state of strain or Uni-axial state of strain**

$$\epsilon_{ij} = \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & -\mu\epsilon_x & 0 \\ 0 & 0 & -\mu\epsilon_x \end{pmatrix} = \begin{pmatrix} \frac{\sigma_x}{E} & 0 & 0 \\ 0 & -\mu \frac{\sigma_x}{E} & 0 \\ 0 & 0 & -\mu \frac{\sigma_x}{E} \end{pmatrix} = \begin{pmatrix} p_y & 0 & 0 \\ 0 & q_y & 0 \\ 0 & 0 & q_y \end{pmatrix}$$

- **2-D Strain** ( $\sigma_z = 0$ )

$$(i) \quad \epsilon_x = \frac{1}{E} [\sigma_x - \mu \sigma_y]$$

$$\epsilon_y = \frac{1}{E} [\sigma_y - \mu \sigma_x]$$

$$\epsilon_z = -\frac{\mu}{E} [\sigma_x + \sigma_y]$$

[Where,  $\epsilon_x, \epsilon_y, \epsilon_z$  are strain component in X, Y, and Z axis respectively]

$$(ii) \quad \sigma_x = \frac{E}{1-\mu^2} [\epsilon_x + \mu \epsilon_y]$$

$$\sigma_y = \frac{E}{1-\mu^2} [\epsilon_y + \mu \epsilon_x]$$

- **3-D Strain**

$$(i) \quad \epsilon_x = \frac{1}{E} [\sigma_x - \mu (\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E} [\sigma_y - \mu (\sigma_z + \sigma_x)]$$

$$\epsilon_z = \frac{1}{E} [\sigma_z - \mu (\sigma_x + \sigma_y)]$$

$$(ii) \quad \sigma_x = \frac{E}{(1+\mu)(1-2\mu)} [(1-\mu)\epsilon_x + \mu(\epsilon_y + \epsilon_z)]$$

## Chapter-2

### Principal Stress and Strain

$$\sigma_y = \frac{E}{(1+\mu)(1-2\mu)} \left[ (1-\mu)\epsilon_y + \mu(\epsilon_z + \epsilon_x) \right]$$

$$\sigma_z = \frac{E}{(1+\mu)(1-2\mu)} \left[ (1-\mu)\epsilon_z + \mu(\epsilon_x + \epsilon_y) \right]$$

**Let us take an example:** At a point in a loaded member, a state of plane stress exists and the strains are  $\epsilon_x = 270 \times 10^{-6}$ ;  $\epsilon_y = -90 \times 10^{-6}$  and  $\epsilon_{xy} = 360 \times 10^{-6}$ . If the elastic constants  $E$ ,  $\mu$  and  $G$  are 200 GPa, 0.25 and 80 GPa respectively.

Determine the normal stress  $\sigma_x$  and  $\sigma_y$  and the shear stress  $\tau_{xy}$  at the point.

**Answer:** We know that

$$\epsilon_x = \frac{1}{E} \{ \sigma_x - \mu \sigma_y \}$$

$$\epsilon_y = \frac{1}{E} \{ \sigma_y - \mu \sigma_x \}$$

$$\epsilon_{xy} = \frac{\tau_{xy}}{G}$$

$$\text{This gives } \sigma_x = \frac{E}{1-\mu^2} \{ \epsilon_x + \mu \epsilon_y \} = \frac{200 \times 10^9}{1-0.25^2} \left[ +270 \times 10^{-6} - 0.25 \times 90 \times 10^{-6} \right] \text{Pa} \\ = 52.8 \text{ MPa (i.e. tensile)}$$

$$\text{and } \sigma_y = \frac{E}{1-\mu^2} \{ \epsilon_y + \mu \epsilon_x \}$$

$$= \frac{200 \times 10^9}{1-0.25^2} \left[ -90 \times 10^{-6} + 0.25 \times 270 \times 10^{-6} \right] \text{Pa} = -4.8 \text{ MPa (i.e. compressive)}$$

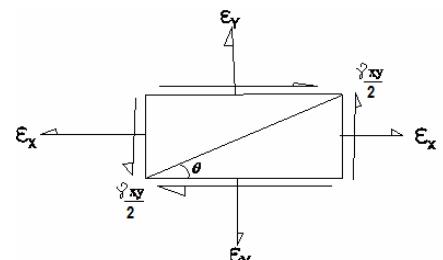
$$\text{and } \tau_{xy} = \epsilon_{xy} \cdot G = 360 \times 10^{-6} \times 80 \times 10^9 \text{ Pa} = 28.8 \text{ MPa}$$

### 2.12 An element subjected to strain components $\epsilon_x, \epsilon_y$ & $\frac{\gamma_{xy}}{2}$

Consider an element as shown in the figure given. The strain component in X-direction is  $\epsilon_x$ , the strain component in Y-direction is  $\epsilon_y$  and the shear strain component is  $\gamma_{xy}$ .

Now consider a plane at an angle  $\theta$  with X-axis in this plane a normal strain  $\epsilon_\theta$  and a shear strain  $\gamma_\theta$ . Then

- $\epsilon_\theta = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$
- $\frac{\gamma_\theta}{2} = -\frac{\epsilon_x - \epsilon_y}{2} \sin 2\theta + \frac{\gamma_{xy}}{2} \cos 2\theta$



## Chapter-2

## Principal Stress and Strain

We may find principal strain and principal plane for strains in the same process which we followed for stress analysis.

In the principal plane shear strain is zero.

Therefore principal strains are

$$\epsilon_{1,2} = \frac{\epsilon_x + \epsilon_y}{2} \pm \sqrt{\left(\frac{\epsilon_x - \epsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

The angle of principal plane

$$\tan 2\theta_p = \frac{\gamma_{xy}}{(\epsilon_x - \epsilon_y)}$$

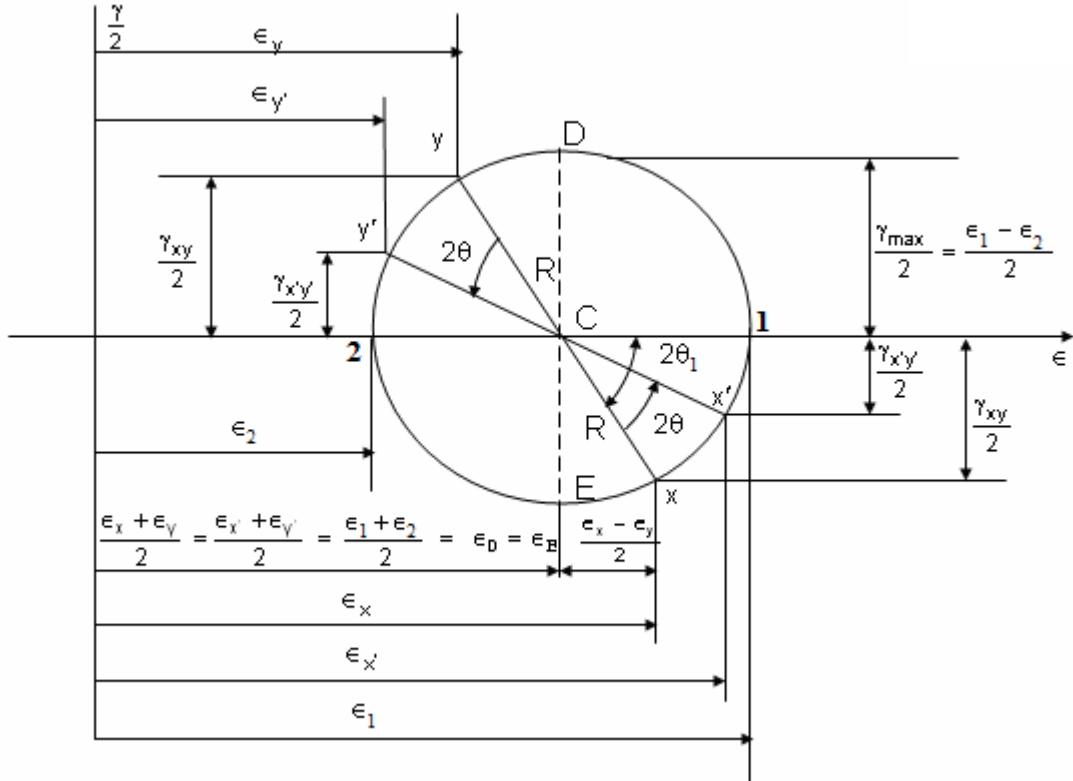
- Maximum shearing strain is equal to the difference between the 2 principal strains i.e

$$(\gamma_{xy})_{\max} = \epsilon_1 - \epsilon_2$$

## Mohr's Circle for circle for Plain Strain

We may draw Mohr's circle for strain following same procedure which we followed for drawing Mohr's circle in stress. Everything will be same and in the place of  $\sigma_x$  write  $\epsilon_x$ , the place of

$\sigma_y$  write  $\epsilon_y$  and in place of  $\tau_{xy}$  write  $\frac{\gamma_{xy}}{2}$ .



## 2.15 Volumetric Strain (Dilation)

- *Rectangular block,*

$$\frac{\Delta V}{V_0} = \epsilon_x + \epsilon_y + \epsilon_z$$

**Proof:** Volumetric strain

$$\begin{aligned} \frac{\Delta V}{V_0} &= \frac{V - V_0}{V_0} \\ &= \frac{L(1 + \epsilon_x) \times L(1 + \epsilon_y) \times L(1 + \epsilon_z) - L^3}{L^3} \\ &= \epsilon_x + \epsilon_y + \epsilon_z \end{aligned}$$

(neglecting second and third order term, as very small )

- *In case of prismatic bar,*

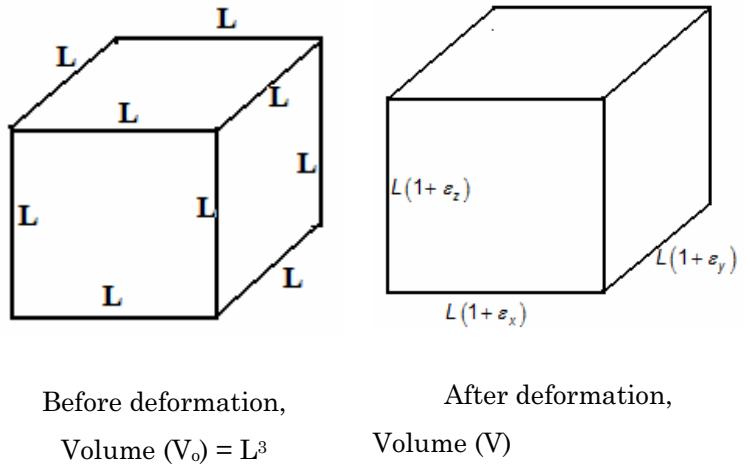
$$\text{Volumetric strain, } \frac{dV}{V} = \epsilon(1 - 2\mu)$$

**Proof:** Before deformation, the volume of the bar,  $V = A \cdot L$

After deformation, the length  $(L') = L(1 + \epsilon)$

and the new cross-sectional area  $(A') = A(1 - \mu\epsilon)^2$

Therefore now volume  $(V') = A' L' = A L (1 + \epsilon)(1 - \mu\epsilon)^2$



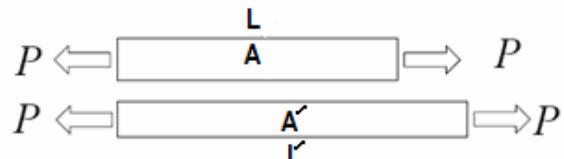
Before deformation,

Volume ( $V_0$ ) =  $L^3$

After deformation,

Volume (V)

$$= L(1 + \epsilon_x) \times L(1 + \epsilon_y) \times L(1 + \epsilon_z)$$



$$P \leftarrow \boxed{A} \rightarrow P$$

$$P \leftarrow \boxed{A'} \rightarrow P$$

$$P \leftarrow \boxed{L'} \rightarrow P$$

## Chapter-2

## Principal Stress and Strain

$$\therefore \frac{\Delta V}{V} = \frac{V' - V}{V} = \frac{AL(1+\varepsilon)(1-\mu\varepsilon)^2 - AL}{AL} = \varepsilon(1-2\mu)$$
$$\frac{\Delta V}{V} = \varepsilon(1-2\mu)$$

- ***Thin Cylindrical vessel***

$$\varepsilon_1 = \text{Longitudinal strain} = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E} = \frac{pr}{2Et} [1 - 2\mu]$$

$$\varepsilon_2 = \text{Circumferential strain} = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E} = \frac{pr}{2Et} [2 - \mu]$$

$$\frac{\Delta V}{V_o} = \varepsilon_1 + 2\varepsilon_2 = \frac{pr}{2Et} [5 - 4\mu]$$

- ***Thin Spherical vessels***

$$\varepsilon = \varepsilon_1 = \varepsilon_2 = \frac{pr}{2Et} [1 - \mu]$$

$$\frac{\Delta V}{V_o} = 3\varepsilon = \frac{3pr}{2Et} [1 - \mu]$$

- ***In case of pure shear***

$$\sigma_x = -\sigma_y = \tau$$

Therefore

$$\varepsilon_x = \frac{\tau}{E} (1 + \mu)$$

$$\varepsilon_y = -\frac{\tau}{E} (1 + \mu)$$

$$\varepsilon_z = 0$$

$$\text{Therefore } \varepsilon_v = \frac{dv}{V} = \varepsilon_x + \varepsilon_y + \varepsilon_z = 0$$

## 2.16 Measurement of Strain

Unlike stress, strain **can** be measured directly. The most common way of measuring strain is by use of the **Strain Gauge**.

### Strain Gauge

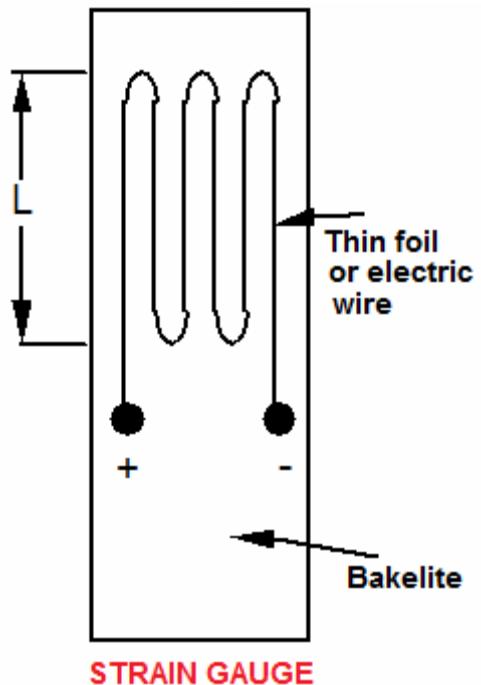
## Chapter-2

## Principal Stress and Strain

A strain gage is a simple device, comprising of a thin electric wire attached to an insulating thin backing material such as a bakelite foil. The foil is exposed to the surface of the specimen on which the strain is to be measured. The thin epoxy layer bonds the gauge to the surface and forces the gauge to shorten or elongate as if it were part of the specimen being strained.

A change in length of the gauge due to longitudinal strain creates a proportional change in the electric resistance, and since a constant current is maintained in the gauge, a proportional change in voltage. ( $V = IR$ ).

The voltage can be easily measured, and through calibration, transformed into the change in length of the original gauge length, i.e. the longitudinal strain along the gauge length.



## Strain Gauge factor (G.F)

$$GF = \frac{\Delta R / R}{\Delta \ell / \ell} = \frac{\Delta R / R}{\epsilon}$$

Measured from Bridge voltage

Given

Calculated

The strain gauge factor relates a change in resistance with strain.

## Strain Rosette

The **strain rosette** is a device used to measure the state of strain at a point in a plane.

It comprises ***three or more*** independent strain gauges, each of which is used to read normal strain at the same point but in a different direction.

The relative orientation between the three gauges is known as  $\alpha$ ,  $\beta$  and  $\delta$

The three measurements of normal strain provide sufficient information for the determination of the complete state of strain at the measured point in 2-D.

We have to find out  $\epsilon_x$ ,  $\epsilon_y$ , and  $\gamma_{xy}$  from measured value  $\epsilon_a$ ,  $\epsilon_b$ , and  $\epsilon_c$

## Chapter-2

## Principal Stress and Strain

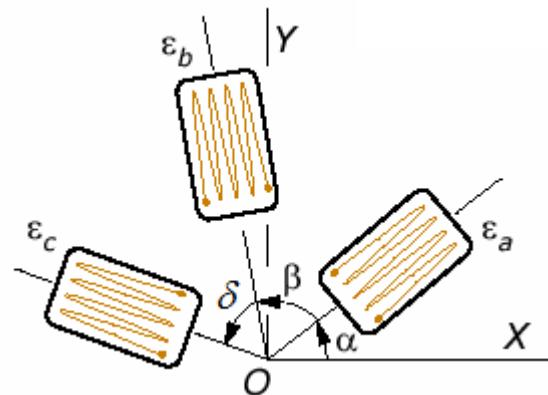
### General arrangement:

The orientation of strain gauges is given in the figure. To relate strain we have to use the following formula.

$$\epsilon_{\theta} = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

We get

$$\epsilon_a = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2\alpha + \frac{\gamma_{xy}}{2} \sin 2\alpha$$



$$\epsilon_b = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2(\alpha + \beta) + \frac{\gamma_{xy}}{2} \sin 2(\alpha + \beta)$$

$$\epsilon_c = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\epsilon_x - \epsilon_y}{2} \cos 2(\alpha + \beta + \delta) + \frac{\gamma_{xy}}{2} \sin 2(\alpha + \beta + \delta)$$

From these three equations and three unknowns we may solve  $\epsilon_x$ ,  $\epsilon_y$ , and  $\gamma_{xy}$

### • Two standard arrangement of the strain rosette are as follows:

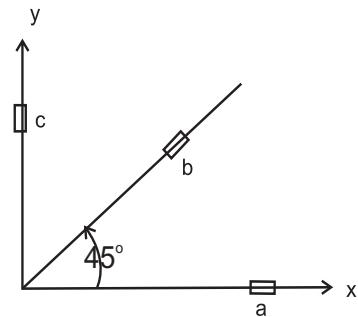
#### (i) 45° strain rosette or Rectangular strain rosette.

In the general arrangement above, put

$$\alpha = 0^\circ; \beta = 45^\circ \text{ and } \delta = 45^\circ$$

Putting the value we get

- $\epsilon_a = \epsilon_x$
- $\epsilon_b = \frac{\epsilon_x + \epsilon_y}{2} + \frac{\gamma_{xy}}{2}$
- $\epsilon_c = \epsilon_y$



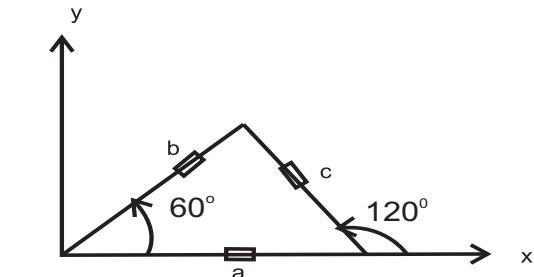
#### (ii) 60° strain rosette or Delta strain rosette

In the general arrangement above, put

$$\alpha = 0^\circ; \beta = 60^\circ \text{ and } \delta = 60^\circ$$

Putting the value we get

- $\epsilon_a = \epsilon_y$
- $\epsilon_b = \frac{\epsilon_x + 3\epsilon_y}{4} + \frac{\sqrt{3}}{4}\gamma_{xy}$
- $\epsilon_c = \frac{\epsilon_x + 3\epsilon_y}{4} - \frac{\sqrt{3}}{4}\gamma_{xy}$



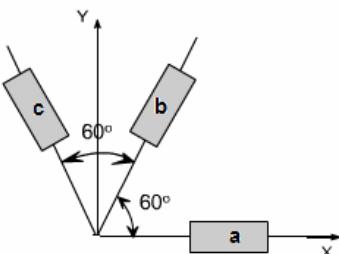
or

Solving above three equations we get

$$\epsilon_x = \epsilon_a$$

$$\epsilon_y = \frac{1}{3}(2\epsilon_b + 2\epsilon_c - \epsilon_a)$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\epsilon_c - \epsilon_b)$$



# OBJECTIVE QUESTIONS (GATE, IES, IAS)

## Previous 20-Years GATE Questions

### Stresses due to Pure Shear

**GATE-1.** A block of steel is loaded by a tangential force on its top surface while the bottom surface is held rigidly. The deformation of the block is due to [GATE-1992]

(a) Shear only      (b) Bending only      (c) Shear and bending      (d) Torsion

**GATE-1. Ans. (a)** It is the definition of shear stress. The force is applied tangentially it is not a point load so you cannot compare it with a cantilever with a point load at its free end.

**GATE-2.** A shaft subjected to torsion experiences a pure shear stress  $\tau$  on the surface. The maximum principal stress on the surface which is at  $45^\circ$  to the axis will have a value [GATE-2003]

(a)  $\tau \cos 45^\circ$       (b)  $2\tau \cos 45^\circ$       (c)  $\tau \cos^2 45^\circ$       (d)  $2\tau \sin 45^\circ \cos 45^\circ$

**GATE-2. Ans. (d)**  $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$

Here  $\sigma_x = \sigma_2 = 0$ ,  $\tau_{xy} = \tau$ ,  $\theta = 45^\circ$

**GATE-3.** The number of components in a stress tensor defining stress at a point in three dimensions is: [GATE-2002]

(a) 3      (b) 4      (c) 6      (d) 9

**GATE-3. Ans. (d)** It is well known that,

$\tau_{xy} = \tau_{yx}$ ,  $\tau_{xz} = \tau_{zx}$  and  $\tau_{yz} = \tau_{zy}$

so that the state of stress at a point is given by six components  $\sigma_x, \sigma_y, \sigma_z$  and  $\tau_{xy}, \tau_{yz}, \tau_{zx}$

### Principal Stress and Principal Plane

**GATE-4.** A body is subjected to a pure tensile stress of 100 units. What is the maximum shear produced in the body at some oblique plane due to the above? [IES-2006]

(a) 100 units      (b) 75 units      (c) 50 units      (d) 0 unit

**GATE-4. Ans. (c)**  $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{100 - 0}{2} = 50$  units.

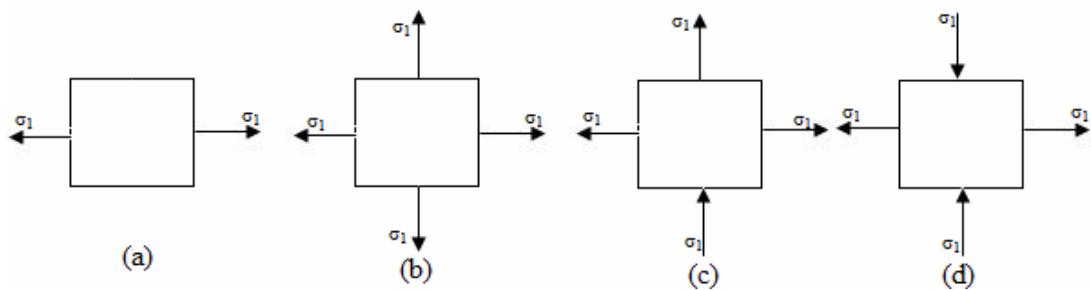
**GATE-5.** In a strained material one of the principal stresses is twice the other. The maximum shear stress in the same case is  $\tau_{\max}$ . Then, what is the value of the maximum principle stress? [IES 2007]

(a)  $\tau_{\max}$       (b)  $2\tau_{\max}$       (c)  $4\tau_{\max}$       (d)  $8\tau_{\max}$

**GATE-5. Ans. (c)**  $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$ ,  $\sigma_1 = 2\sigma_2$  or  $\tau_{\max} = \frac{\sigma_2}{2}$  or  $\sigma_2 = 2\tau_{\max}$  or  $\sigma_1 = 2\sigma_2 = 4\tau_{\max}$

**GATE-6.** A material element subjected to a plane state of stress such that the maximum shear stress is equal to the maximum tensile stress, would correspond to

[IAS-1998]



GATE-6. Ans. (d)  $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_1 - (-\sigma_1)}{2} = \sigma_1$

GATE-7. A solid circular shaft is subjected to a maximum shearing stress of 140 MPa. The magnitude of the maximum normal stress developed in the shaft is:

[IAS-1995]

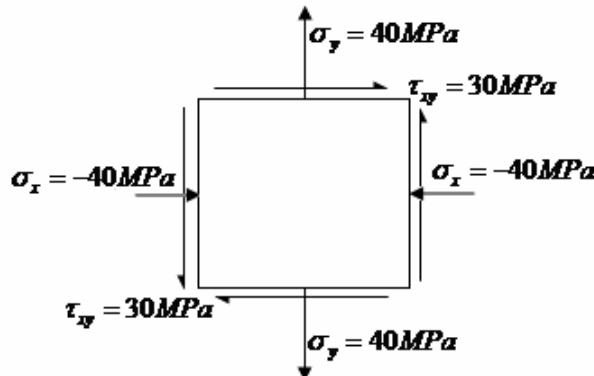
(a) 140 MPa (b) 80 MPa (c) 70 MPa (d) 60 MPa

GATE-7. Ans. (a)  $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$  Maximum normal stress will developed if  $\sigma_1 = -\sigma_2 = \sigma$

GATE-8. The state of stress at a point in a loaded member is shown in the figure. The magnitude of maximum shear stress is [1 MPa = 10 kg/cm<sup>2</sup>]

[IAS 1994]

(a) 10 MPa (b) 30 MPa (c) 50 MPa (d) 100 MPa



GATE-8. Ans. (c)  $\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-40 - 40}{2}\right)^2 + 30^2} = 50 \text{ MPa}$

GATE-9. A solid circular shaft of diameter 100 mm is subjected to an axial stress of 50 MPa. It is further subjected to a torque of 10 kNm. The maximum principal stress experienced on the shaft is closest to

[GATE-2008]

(a) 41 MPa (b) 82 MPa (c) 164 MPa (d) 204 MPa

GATE-9. Ans. (b) Shear Stress ( $\tau$ ) =  $\frac{16T}{\pi d^3} = \frac{16 \times 10000}{\pi \times (0.1)^3} \text{ Pa} = 50.93 \text{ MPa}$

Maximum principal Stress =  $\frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2} = 82 \text{ MPa}$

GATE-10. In a bi-axial stress problem, the stresses in x and y directions are ( $\sigma_x = 200 \text{ MPa}$  and  $\sigma_y = 100 \text{ MPa}$ ). The maximum principal stress in MPa, is:

[GATE-2000]

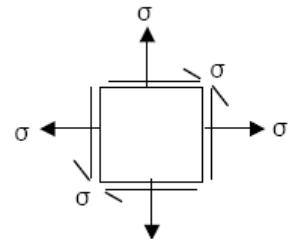
(a) 50 (b) 100 (c) 150 (d) 200

GATE-10. Ans. (d)  $\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \text{if } \tau_{xy} \neq 0$

$$= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2} = \sigma_x$$

**GATE-11.** The maximum principle stress for the stress state shown in the figure is

(a)  $\sigma$  (b)  $2\sigma$   
(c)  $3\sigma$  (d)  $1.5\sigma$



[GATE-2001]

**GATE-11. Ans. (b)**  $\sigma_x = \sigma, \sigma_y = \sigma, \tau_{xy} = \sigma$

$$\therefore (\sigma_1)_{\max} = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{\sigma + \sigma}{2} + \sqrt{(0)^2 + \sigma^2} = 2\sigma$$

**GATE-12.** The normal stresses at a point are  $\sigma_x = 10$  MPa and,  $\sigma_y = 2$  MPa; the shear stress at this point is 4 MPa. The maximum principal stress at this point is:

[GATE-1998]

(a) 16 MPa (b) 14 MPa (c) 11 MPa (d) 10 MPa

$$\text{GATE-12. Ans. (c)} \quad \sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \frac{10 + 2}{2} + \sqrt{\left(\frac{10 - 2}{2}\right)^2 + 4^2} = 11.66 \text{ MPa}$$

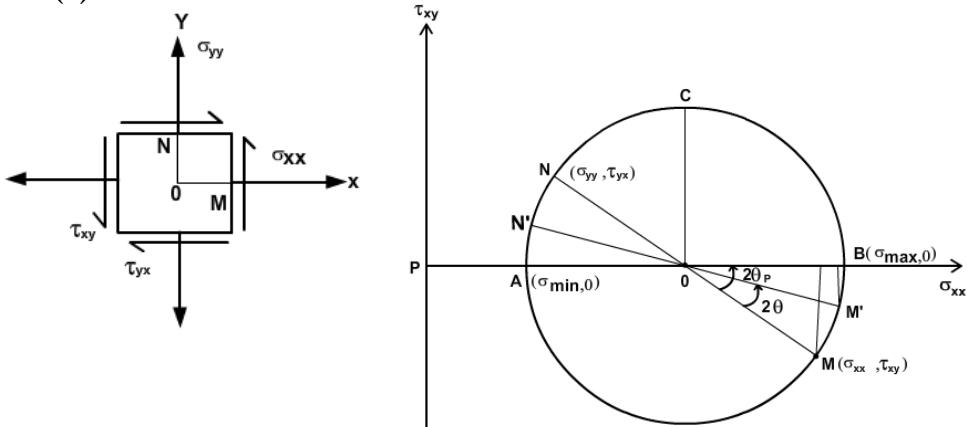
**GATE-13.** In a Mohr's circle, the radius of the circle is taken as:

[IES-2006; GATE-1993]

(a)  $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$  (b)  $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$   
(c)  $\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - (\tau_{xy})^2}$  (d)  $\sqrt{(\sigma_x - \sigma_y)^2 + (\tau_{xy})^2}$

Where,  $\sigma_x$  and  $\sigma_y$  are normal stresses along x and y directions respectively and  $\tau_{xy}$  is the shear stress.

**GATE-13. Ans. (a)**



**GATE-14.** A two dimensional fluid element rotates like a rigid body. At a point within the element, the pressure is 1 unit. Radius of the Mohr's circle, characterizing the state of stress at that point, is:

[GATE-2008]

(a) 0.5 unit (b) 0 unit (c) 1 unit (d) 2 units

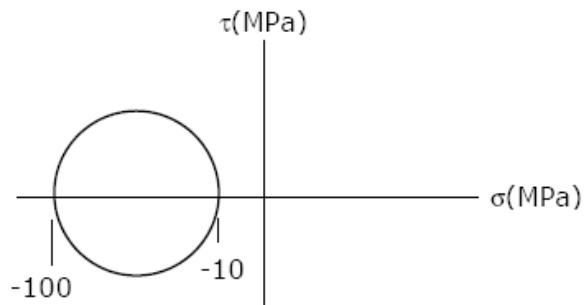
**GATE-14. Ans. (b)**

## Chapter-2

## Principal Stress and Strain

**GATE-15.** The Mohr's circle of plane stress for a point in a body is shown. The design is to be done on the basis of the maximum shear stress theory for yielding. Then, yielding will just begin if the designer chooses a ductile material whose yield strength is:

(a) 45 MPa (b) 50 MPa  
(c) 90 MPa (d) 100 MPa



[GATE-2005]

### GATE-15. Ans. (c)

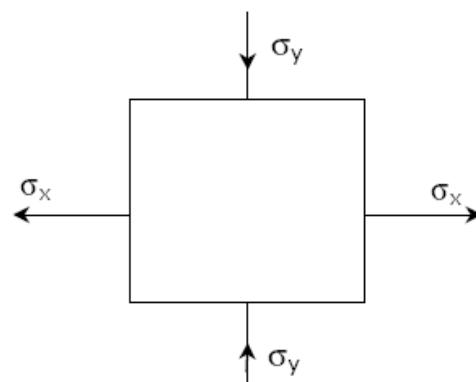
$$\text{Given } \sigma_1 = -10 \text{ MPa}, \quad \sigma_2 = -100 \text{ MPa}$$

$$\text{Maximum shear stress theory give } \tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_y}{2}$$

$$\text{or } \sigma_1 - \sigma_2 = \sigma_y \quad \Rightarrow \sigma_y = -10 - (-100) = 90 \text{ MPa}$$

**GATE-16.** The figure shows the state of stress at a certain point in a stressed body. The magnitudes of normal stresses in the x and y direction are 100 MPa and 20 MPa respectively. The radius of Mohr's stress circle representing this state of stress is:

(a) 120 (b) 80  
(c) 60 (d) 40



[GATE-2004]

### GATE-16. Ans. (c)

$$\sigma_x = 100 \text{ MPa}, \quad \sigma_y = -20 \text{ MPa}$$

$$\text{Radius of Mohr's circle} = \frac{\sigma_x - \sigma_y}{2} = \frac{100 - (-20)}{2} = 60$$

Data for Q17-Q18 are given below. Solve the problems and choose correct answers.

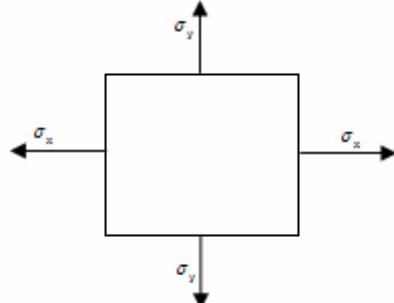
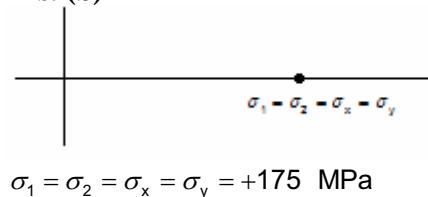
[GATE-2003]

The state of stress at a point "P" in a two dimensional loading is such that the Mohr's circle is a point located at 175 MPa on the positive normal stress axis.

**GATE-17.** Determine the maximum and minimum principal stresses respectively from the Mohr's circle

(a) +175 MPa, -175 MPa (b) +175 MPa, +175 MPa  
(c) 0, -175 MPa (d) 0, 0

### GATE-17. Ans. (b)



**GATE-18.** Determine the directions of maximum and minimum principal stresses at the point "P" from the Mohr's circle

Page 88 of 429

[GATE-2003]

**Chapter-2****Principal Stress and Strain**

(a) 0, 90°

(b) 90°, 0

(c) 45°, 135°

(d) All directions

**GATE-18. Ans. (d)** From the Mohr's circle it will give all directions.**Principal strains****GATE-19.** If the two principal strains at a point are  $1000 \times 10^{-6}$  and  $-600 \times 10^{-6}$ , then the maximum shear strain is: [GATE-1996](a)  $800 \times 10^{-6}$ (b)  $500 \times 10^{-6}$ (c)  $1600 \times 10^{-6}$ (d)  $200 \times 10^{-6}$ **GATE-19. Ans. (c)** Shear strain  $\epsilon_{\max} - \epsilon_{\min} = \{1000 - (-600)\} \times 10^{-6} = 1600 \times 10^{-6}$ **Previous 20-Years IES Questions****Stresses due to Pure Shear****IES-1.** If a prismatic bar be subjected to an axial tensile stress  $\sigma$ , then shear stress induced on a plane inclined at  $\theta$  with the axis will be: [IES-1992](a)  $\frac{\sigma}{2} \sin 2\theta$ (b)  $\frac{\sigma}{2} \cos 2\theta$ (c)  $\frac{\sigma}{2} \cos^2 \theta$ (d)  $\frac{\sigma}{2} \sin^2 \theta$ **IES-1. Ans. (a)****IES-2.** In the case of bi-axial state of normal stresses, the normal stress on  $45^\circ$  plane is equal to [IES-1992]

(a) The sum of the normal stresses

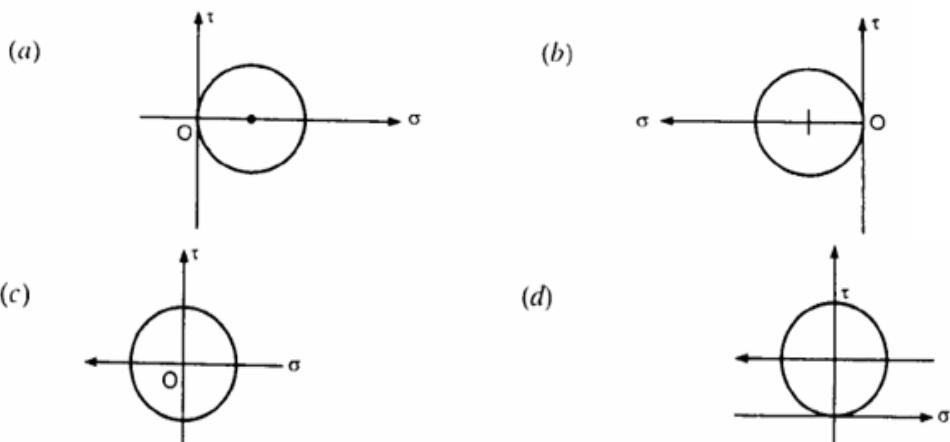
(b) Difference of the normal stresses

(c) Half the sum of the normal stresses

(d) Half the difference of the normal stresses

**IES-2. Ans. (c)**  $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$ 

At  $\theta = 45^\circ$  and  $\tau_{xy} = 0$ ;  $\sigma_n = \frac{\sigma_x + \sigma_y}{2}$

**IES-3.** In a two-dimensional problem, the state of pure shear at a point is characterized by [IES-2001](a)  $\epsilon_x = \epsilon_y$  and  $\gamma_{xy} = 0$ (b)  $\epsilon_x = -\epsilon_y$  and  $\gamma_{xy} \neq 0$ (c)  $\epsilon_x = 2\epsilon_y$  and  $\gamma_{xy} \neq 0$ (d)  $\epsilon_x = 0.5\epsilon_y$  and  $\gamma_{xy} = 0$ **IES-3. Ans. (b)****IES-4.** Which one of the following Mohr's circles represents the state of pure shear? [IES-2000]**IES-4. Ans. (c)**

**Chapter-2****Principal Stress and Strain**

**IES-5.** For the state of stress of pure shear  $\tau$  the strain energy stored per unit volume in the elastic, homogeneous isotropic material having elastic constants E and  $\nu$  will be: [IES-1998]

$$(a) \frac{\tau^2}{E}(1+\nu) \quad (b) \frac{\tau^2}{2E}(1+\nu) \quad (c) \frac{2\tau^2}{E}(1+\nu) \quad (d) \frac{\tau^2}{2E}(2+\nu)$$

**IES-5. Ans. (a)**  $\sigma_1 = \tau, \sigma_2 = -\tau, \sigma_3 = 0$

$$U = \frac{1}{2E} \left[ \tau^2 + (-\tau)^2 - 2\mu\tau(-\tau) \right] V = \frac{1+\mu}{E} \tau^2 V$$

**IES-6.** Assertion (A): If the state at a point is pure shear, then the principal planes through that point making an angle of  $45^\circ$  with plane of shearing stress carries principal stresses whose magnitude is equal to that of shearing stress.

Reason (R): Complementary shear stresses are equal in magnitude, but opposite in direction. [IES-1996]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-6. Ans. (b)**

**IES-7.** Assertion (A): Circular shafts made of brittle material fail along a helicoidally surface inclined at  $45^\circ$  to the axis (artery point) when subjected to twisting moment. [IES-1995]

Reason (R): The state of pure shear caused by torsion of the shaft is equivalent to one of tension at  $45^\circ$  to the shaft axis and equal compression in the perpendicular direction.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-7. Ans. (a)** Both A and R are true and R is correct explanation for A.

**IES-8.** A state of pure shear in a biaxial state of stress is given by [IES-1994]

$$(a) \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \quad (b) \begin{pmatrix} \sigma_1 & 0 \\ 0 & -\sigma_1 \end{pmatrix} \quad (c) \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{pmatrix} \quad (d) \text{None of the above}$$

**IES-8. Ans. (b)**  $\sigma_1 = \tau, \sigma_2 = -\tau, \sigma_3 = 0$

**IES-9.** The state of plane stress in a plate of 100 mm thickness is given as [IES-2000]  $\sigma_{xx} = 100 \text{ N/mm}^2, \sigma_{yy} = 200 \text{ N/mm}^2, \text{Young's modulus} = 300 \text{ N/mm}^2, \text{Poisson's ratio} = 0.3$ . The stress developed in the direction of thickness is:

- (a) Zero
- (b)  $90 \text{ N/mm}^2$
- (c)  $100 \text{ N/mm}^2$
- (d)  $200 \text{ N/mm}^2$

**IES-9. Ans. (a)**

**IES-10.** The state of plane stress at a point is described by  $\sigma_x = \sigma_y = \sigma$  and  $\tau_{xy} = 0$ . The normal stress on the plane inclined at  $45^\circ$  to the x-plane will be: [IES-1998]

- (a)  $\sigma$
- (b)  $\sqrt{2}\sigma$
- (c)  $\sqrt{3}\sigma$
- (d)  $2\sigma$

**IES-10. Ans. (a)**  $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$

**IES-11.** Consider the following statements: [IES-1996, 1998]

State of stress in two dimensions at a point in a loaded component can be completely specified by indicating the normal and shear stresses on

1. A plane containing the point
2. Any two planes passing through the point
3. Two mutually perpendicular planes passing through the point

## Chapter-2

## Principal Stress and Strain

Of these statements

(a) 1, and 3 are correct  
(c) 1 alone is correct

(b) 2 alone is correct  
(d) 3 alone is correct

IES-11. Ans. (d)

## Principal Stress and Principal Plane

IES-12. A body is subjected to a pure tensile stress of 100 units. What is the maximum shear produced in the body at some oblique plane due to the above? [IES-2006]

(a) 100 units (b) 75 units (c) 50 units (d) 0 unit

IES-12. Ans. (c)  $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{100 - 0}{2} = 50$  units.

IES-13. In a strained material one of the principal stresses is twice the other. The maximum shear stress in the same case is  $\tau_{\max}$ . Then, what is the value of the maximum principle stress? [IES 2007]

(a)  $\tau_{\max}$  (b)  $2\tau_{\max}$  (c)  $4\tau_{\max}$  (d)  $8\tau_{\max}$

IES-13. Ans. (c)  $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$ ,  $\sigma_1 = 2\sigma_2$  or  $\tau_{\max} = \frac{\sigma_2}{2}$  or  $\sigma_2 = 2\tau_{\max}$  or  $\sigma_1 = 2\sigma_2 = 4\tau_{\max}$

IES-14. In a strained material, normal stresses on two mutually perpendicular planes are  $\sigma_x$  and  $\sigma_y$  (both alike) accompanied by a shear stress  $\tau_{xy}$ . One of the principal stresses will be zero, only if [IES-2006]

(a)  $\tau_{xy} = \frac{\sigma_x \times \sigma_y}{2}$  (b)  $\tau_{xy} = \sigma_x \times \sigma_y$  (c)  $\tau_{xy} = \sqrt{\sigma_x \times \sigma_y}$  (d)  $\tau_{xy} = \sqrt{\sigma_x^2 + \sigma_y^2}$

IES-14. Ans. (c)  $\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

if  $\sigma_2 = 0 \Rightarrow \frac{\sigma_x + \sigma_y}{2} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

or  $\left(\frac{\sigma_x + \sigma_y}{2}\right)^2 = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$  or  $\tau_{xy} = \sqrt{\sigma_x \times \sigma_y}$

IES-15. The principal stresses  $\sigma_1$ ,  $\sigma_2$  and  $\sigma_3$  at a point respectively are 80 MPa, 30 MPa and -40 MPa. The maximum shear stress is: [IES-2001]

(a) 25 MPa (b) 35 MPa (c) 55 MPa (d) 60 MPa

IES-15. Ans. (d)  $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{80 - (-40)}{2} = 60$  MPa

IES-16. Plane stress at a point in a body is defined by principal stresses  $3\sigma$  and  $\sigma$ . The ratio of the normal stress to the maximum shear stresses on the plane of maximum shear stress is: [IES-2000]

(a) 1 (b) 2 (c) 3 (d) 4

IES-16. Ans. (b)  $\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \Rightarrow \theta = 0$

$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{3\sigma - \sigma}{2} = \sigma$

Major principal stress on the plane of maximum shear =  $\sigma_1 = \frac{3\sigma + \sigma}{2} = 2\sigma$

## Chapter-2

## Principal Stress and Strain

IES-17. Principal stresses at a point in plane stressed element are  $\sigma_x = \sigma_y = 500 \text{ kg/cm}^2$ .

Normal stress on the plane inclined at  $45^\circ$  to x-axis will be: [IES-1993]

(a) 0 (b)  $500 \text{ kg/cm}^2$  (c)  $707 \text{ kg/cm}^2$  (d)  $1000 \text{ kg/cm}^2$

IES-17. Ans. (b) When stresses are alike, then normal stress  $\sigma_n$  on plane inclined at angle  $45^\circ$  is

$$\sigma_n = \sigma_y \cos^2 \theta + \sigma_x \sin^2 \theta = \sigma_y \left( \frac{1}{\sqrt{2}} \right)^2 + \sigma_x \left( \frac{1}{\sqrt{2}} \right)^2 = 500 \left[ \frac{1}{2} + \frac{1}{2} \right] = 500 \text{ kg/cm}^2$$

IES-18. If the principal stresses corresponding to a two-dimensional state of stress are  $\sigma_1$  and  $\sigma_2$  is greater than  $\sigma_2$  and both are tensile, then which one of the following would be the correct criterion for failure by yielding, according to the maximum shear stress criterion? [IES-1993]

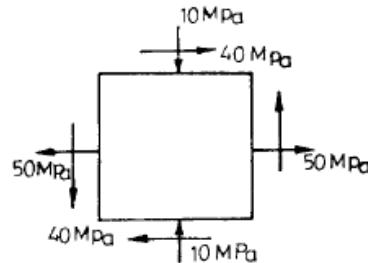
(a)  $\frac{(\sigma_1 - \sigma_2)}{2} = \pm \frac{\sigma_{yp}}{2}$  (b)  $\frac{\sigma_1}{2} = \pm \frac{\sigma_{yp}}{2}$  (c)  $\frac{\sigma_2}{2} = \pm \frac{\sigma_{yp}}{2}$  (d)  $\sigma_1 = \pm 2\sigma_{yp}$

IES-18. Ans. (a)

IES-19. For the state of plane stress.

Shown the maximum and minimum principal stresses are:

(a) 60 MPa and 30 MPa  
(b) 50 MPa and 10 MPa  
(c) 40 MPa and 20 MPa  
(d) 70 MPa and 30 MPa



[IES-1992]

$$\text{IES-19. Ans. (d)} \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$\sigma_{1,2} = \frac{50 + (-10)}{2} \pm \sqrt{\left( \frac{50 + 10}{2} \right)^2 + 40^2}$$

$$\sigma_{\max} = 70 \text{ and } \sigma_{\min} = -30$$

IES-20. Normal stresses of equal magnitude  $p$ , but of opposite signs, act at a point of a strained material in perpendicular direction. What is the magnitude of the resultant normal stress on a plane inclined at  $45^\circ$  to the applied stresses? [IES-2005]

(a)  $2p$  (b)  $p/2$  (c)  $p/4$  (d) Zero

$$\text{IES-20. Ans. (d)} \sigma_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$$

$$\sigma_n = \frac{P - P}{2} + \frac{P + P}{2} \cos 2 \times 45^\circ = 0$$

IES-21. A plane stressed element is subjected to the state of stress given by  $\sigma_x = \tau_{xy} = 100 \text{ kgf/cm}^2$  and  $\sigma_y = 0$ . Maximum shear stress in the element is equal to [IES-1997]

(a)  $50\sqrt{3} \text{ kgf/cm}^2$  (b)  $100 \text{ kgf/cm}^2$  (c)  $50\sqrt{5} \text{ kgf/cm}^2$  (d)  $150 \text{ kgf/cm}^2$

## Chapter-2

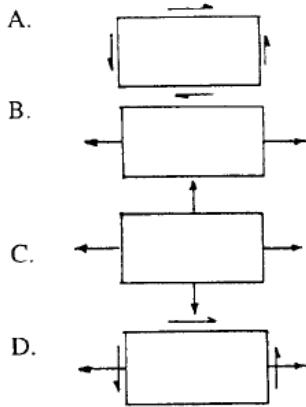
## Principal Stress and Strain

IES-21. Ans. (c)  $(\sigma)_{1,2} = \frac{\sigma_x + 0}{2} \pm \sqrt{\left(\frac{\sigma_x + 0}{2}\right)^2 + \tau_{xy}^2} = 50 \mp 50\sqrt{5}$

$$\text{Maximum shear stress} = \frac{(\sigma)_1 - (\sigma)_2}{2} = 50\sqrt{5}$$

IES-22. Match List I with List II and select the correct answer, using the codes given below the lists: [IES-1995]

### List I(State of stress)



### List II(Kind of loading)

- Combined bending and torsion of circular shaft.
- Torsion of circular shaft.
- Thin cylinder subjected to internal pressure.
- Tie bar subjected to tensile force.

Codes:	A	B	C	D	A	B	C	D
(a)	1	2	3	4	(b)	2	3	4
(c)	2	4	3	1	(d)	3	4	1

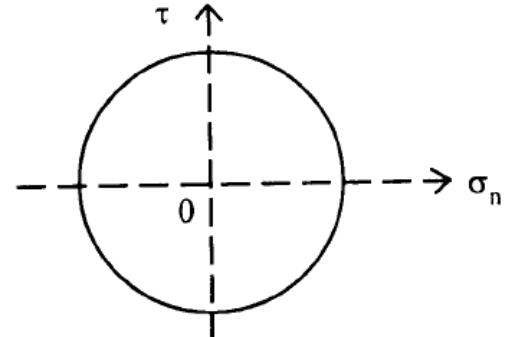
IES-22. Ans. (c)

## Mohr's circle

IES-23. Consider the Mohr's circle shown above:

What is the state of stress represented by this circle?

- $\sigma_x = \sigma_y \neq 0, \tau_{xy} = 0$
- $\sigma_x + \sigma_y = 0, \tau_{xy} \neq 0$
- $\sigma_x = 0, \sigma_y = \tau_{xy} \neq 0$
- $\sigma_x \neq 0, \sigma_y = \tau_{xy} = 0$



[IES-2008]

IES-23. Ans. (b) It is a case of pure shear. Just put  $\sigma_1 = -\sigma_2$

IES-24. For a general two dimensional stress system, what are the coordinates of the centre of Mohr's circle?

$$(a) \frac{\sigma_x - \sigma_y}{2}, 0 \quad (b) 0, \frac{\sigma_x + \sigma_y}{2} \quad (c) \frac{\sigma_x + \sigma_y}{2}, 0 \quad (d) 0, \frac{\sigma_x - \sigma_y}{2}$$

IES-24. Ans. (c)

IES-25. In a Mohr's circle, the radius of the circle is taken as: [IES-2006; GATE-1993]

$$(a) \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2} \quad (b) \sqrt{\frac{(\sigma_x - \sigma_y)^2}{2} + (\tau_{xy})^2}$$

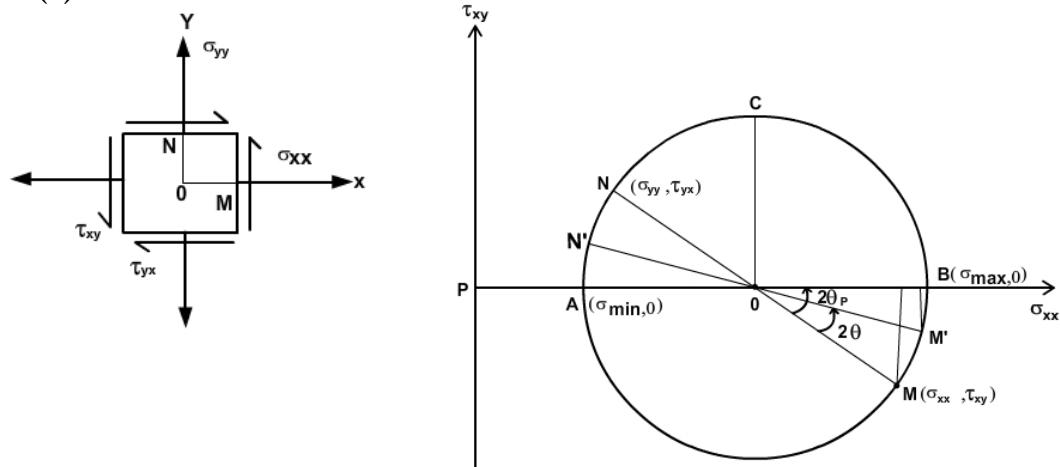
$$(c) \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 - (\tau_{xy})^2} \quad (d) \sqrt{(\sigma_x - \sigma_y)^2 + (\tau_{xy})^2}$$

## Chapter-2

## Principal Stress and Strain

Where,  $\sigma_x$  and  $\sigma_y$  are normal stresses along x and y directions respectively and  $\tau_{xy}$  is the shear stress.

IES-25. Ans. (a)

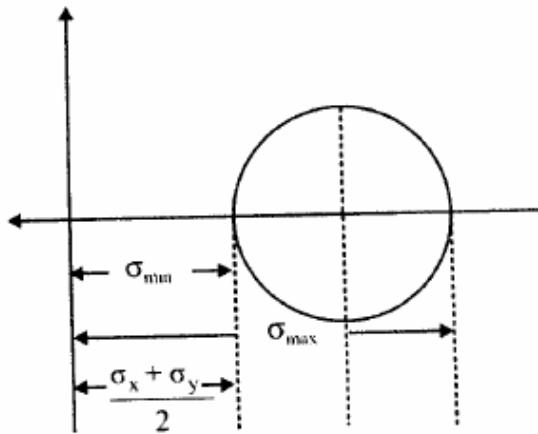


IES-26. Maximum shear stress in a Mohr's Circle

[IES- 2008]

- (a) Is equal to radius of Mohr's circle
- (b) Is greater than radius of Mohr's circle
- (c) Is less than radius of Mohr's circle
- (d) Could be any of the above

IES-26. Ans. (a)



$$\left( \sigma_{x'} - \frac{\sigma_x + \sigma_y}{2} \right)^2 + \tau_{x'y'}^2 = \left( \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \right)^2$$

∴ Radius of the Mohr Circle

$$\sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$\therefore \sigma_t = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$\Rightarrow \tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = r \quad \Rightarrow \tau_{\max} = \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

IES-27. At a point in two-dimensional stress system  $\sigma_x = 100$  N/mm<sup>2</sup>,  $\sigma_y = \tau_{xy} = 40$  N/mm<sup>2</sup>. What is the radius of the Mohr circle for stress drawn with a scale of: 1 cm = 10 N/mm<sup>2</sup>?

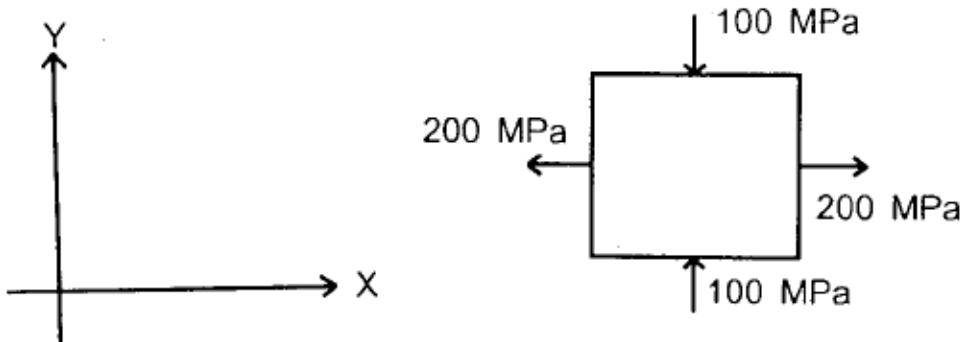
[IES-2005]

- (a) 3 cm
- (b) 4 cm
- (c) 5 cm
- (d) 6 cm

IES-27. Ans. (c) Radius of the Mohr circle

$$= \left[ \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \right] / 10 = \left[ \sqrt{\left( \frac{100 - 40}{2} \right)^2 + 40^2} \right] / 10 = 50 / 10 = 5 \text{ cm}$$

IES-28. Consider a two dimensional state of stress given for an element as shown in the diagram given below: [IES-2004]



What are the coordinates of the centre of Mohr's circle?

(a) (0, 0) (b) (100, 200) (c) (200, 100) (d) (50, 0)

IES-28. Ans. (d) Centre of Mohr's circle is  $\left( \frac{\sigma_x + \sigma_y}{2}, 0 \right) = \left( \frac{200 - 100}{2}, 0 \right) = (50, 0)$

IES-29. Two-dimensional state of stress at a point in a plane stressed element is represented by a Mohr circle of zero radius. Then both principal stresses

(a) Are equal to zero [IES-2003]  
 (b) Are equal to zero and shear stress is also equal to zero  
 (c) Are of equal magnitude but of opposite sign  
 (d) Are of equal magnitude and of same sign

IES-29. Ans. (d)

IES-30. Assertion (A): Mohr's circle of stress can be related to Mohr's circle of strain by some constant of proportionality. [IES-2002]

Reason (R): The relationship is a function of yield stress of the material.

(a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is NOT the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

IES-30. Ans. (c)

IES-31. When two mutually perpendicular principal stresses are unequal but like, the maximum shear stress is represented by [IES-1994]

(a) The diameter of the Mohr's circle  
 (b) Half the diameter of the Mohr's circle  
 (c) One-third the diameter of the Mohr's circle  
 (d) One-fourth the diameter of the Mohr's circle

IES-31. Ans. (b)

IES-32. State of stress in a plane element is shown in figure I. Which one of the following figures-II is the correct sketch of Mohr's circle of the state of stress? [IES-1993, 1996]

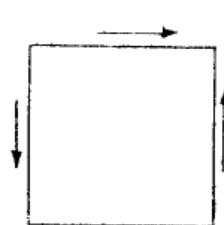


Figure-I

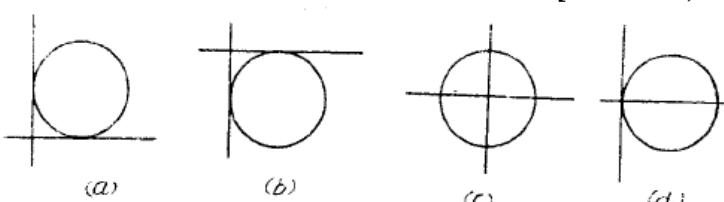


Figure-II

## Strain

IES-33. A point in a two dimensional state of strain is subjected to pure shearing strain of magnitude  $\gamma_{xy}$  radians. Which one of the following is the maximum principal strain?  
 (a)  $\gamma_{xy}$  (b)  $\gamma_{xy}/\sqrt{2}$  (c)  $\gamma_{xy}/2$  (d)  $2\gamma_{xy}$  [IES-2008]

IES-33. Ans. (c)

IES-34. Assertion (A): A plane state of stress does not necessarily result into a plane state of strain as well. [IES-1996]

Reason (R): Normal stresses acting along X and Y directions will also result into normal strain along the Z-direction.

(a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is NOT the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

IES-34. Ans. (a)

## Principal strains

IES-35. Principal strains at a point are  $100 \times 10^{-6}$  and  $-200 \times 10^{-6}$ . What is the maximum shear strain at the point?  
 [IES-2006]

(a)  $300 \times 10^{-6}$  (b)  $200 \times 10^{-6}$  (c)  $150 \times 10^{-6}$  (d)  $100 \times 10^{-6}$

IES-35. Ans. (a)  $\gamma_{\max} = \varepsilon_1 - \varepsilon_2 = 100 - (-200) \times 10^{-6} = 300 \times 10^{-6}$

don't confuse with Maximum Shear stress ( $\tau_{\max}$ ) =  $\frac{\sigma_1 - \sigma_2}{2}$

in strain  $\frac{\gamma_{xy}}{2} = \frac{\varepsilon_1 - \varepsilon_2}{2}$  and  $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$  that is the difference.

IES-36. The principal strains at a point in a body, under biaxial state of stress, are  $1000 \times 10^{-6}$  and  $-600 \times 10^{-6}$ . What is the maximum shear strain at that point?  
 [IES-2009]

(a)  $200 \times 10^{-6}$  (b)  $800 \times 10^{-6}$  (c)  $1000 \times 10^{-6}$  (d)  $1600 \times 10^{-6}$

IES-36. Ans. (d)

$$\frac{\varepsilon_x - \varepsilon_y}{2} = \frac{\phi_{xy}}{2} \Rightarrow \phi_{xy} = \varepsilon_x - \varepsilon_y = 1000 \times 10^{-6} - (-600 \times 10^{-6}) = 1600 \times 10^{-6}$$

IES-37. The number of strain readings (using strain gauges) needed on a plane surface to determine the principal strains and their directions is:  
 [IES-1994]

(a) 1 (b) 2 (c) 3 (d) 4

IES-37. Ans. (c) Three strain gauges are needed on a plane surface to determine the principal strains and their directions.

## Principal strain induced by principal stress

IES-38. The principal stresses at a point in two dimensional stress system are  $\sigma_1$  and  $\sigma_2$  and corresponding principal strains are  $\varepsilon_1$  and  $\varepsilon_2$ . If E and  $\nu$  denote Young's modulus and Poisson's ratio, respectively, then which one of the following is correct?  
 [IES-2008]

(a)  $\sigma_1 = E\varepsilon_1$  (b)  $\sigma_1 = \frac{E}{1-\nu^2} [\varepsilon_1 + \nu\varepsilon_2]$

(c)  $\sigma_1 = \frac{E}{1-\nu^2} [\varepsilon_1 - \nu\varepsilon_2]$  (d)  $\sigma_1 = E [\varepsilon_1 - \nu\varepsilon_2]$

## Chapter-2

## Principal Stress and Strain

IES-38. Ans. (b)  $\varepsilon_1 = \frac{\sigma_1}{E} - \mu \frac{\sigma_2}{E}$  and  $\varepsilon_2 = \frac{\sigma_2}{E} - \mu \frac{\sigma_1}{E}$  From these two equations eliminate  $\sigma_2$ .

IES-39. Assertion (A): Mohr's construction is possible for stresses, strains and area moment of inertia. [IES-2009]

Reason (R): Mohr's circle represents the transformation of second-order tensor.

- (a) Both A and R are individually true and R is the correct explanation of A.
- (b) Both A and R are individually true but R is NOT the correct explanation of A.
- (c) A is true but R is false.
- (d) A is false but R is true.

IES-39. Ans. (a)

## Previous 20-Years IAS Questions

### Stresses due to Pure Shear

IAS-1. On a plane, resultant stress is inclined at an angle of  $45^\circ$  to the plane. If the normal stress is  $100 \text{ N/mm}^2$ , the shear stress on the plane is: [IAS-2003]

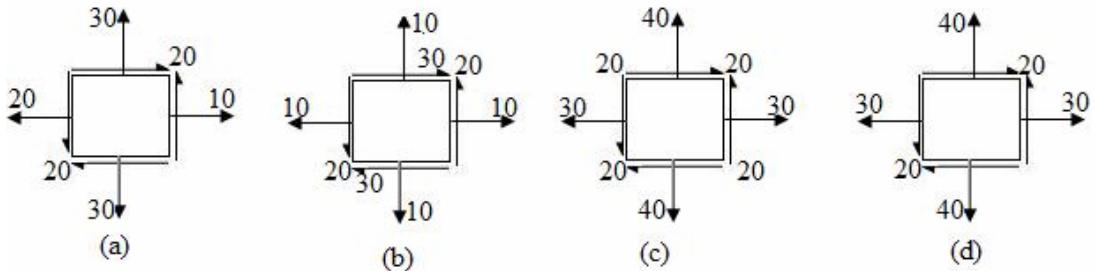
- (a)  $71.5 \text{ N/mm}^2$
- (b)  $100 \text{ N/mm}^2$
- (c)  $86.6 \text{ N/mm}^2$
- (d)  $120.8 \text{ N/mm}^2$

IAS-1. Ans. (b) We know  $\sigma_n = \sigma \cos^2 \theta$  and  $\tau = \sigma \sin \theta \cos \theta$

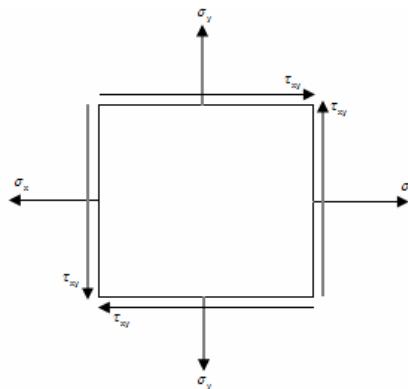
$$100 = \sigma \cos^2 45^\circ \text{ or } \sigma = 200$$

$$\tau = 200 \sin 45^\circ \cos 45^\circ = 100$$

IAS-2. Biaxial stress system is correctly shown in

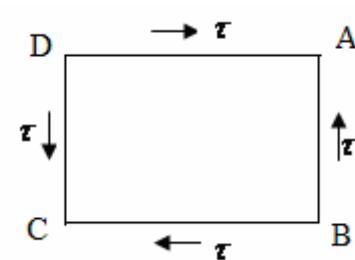


IAS-2. Ans. (c)



IAS-3. The complementary shear stresses of intensity  $\tau$  are induced at a point in the material, as shown in the figure. Which one of the following is the correct set of orientations of principal planes with respect to AB?

- (a)  $30^\circ$  and  $120^\circ$
- (b)  $45^\circ$  and  $135^\circ$
- (c)  $60^\circ$  and  $150^\circ$
- (d)  $75^\circ$  and  $165^\circ$



[IAS-1998]

## Chapter-2

## Principal Stress and Strain

IAS-3. Ans. (b) It is a case of pure shear so principal planes will be along the diagonal.

IAS-4. A uniform bar lying in the  $x$ -direction is subjected to pure bending. Which one of the following tensors represents the strain variations when bending moment is about the  $z$ -axis ( $p$ ,  $q$  and  $r$  constants)? [IAS-2001]

$$(a) \begin{pmatrix} py & 0 & 0 \\ 0 & qy & 0 \\ 0 & 0 & ry \end{pmatrix}$$

$$(b) \begin{pmatrix} py & 0 & 0 \\ 0 & qy & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$(c) \begin{pmatrix} py & 0 & 0 \\ 0 & py & 0 \\ 0 & 0 & py \end{pmatrix}$$

$$(d) \begin{pmatrix} py & 0 & 0 \\ 0 & qy & 0 \\ 0 & 0 & qy \end{pmatrix}$$

IAS-4. Ans. (d) Stress in  $x$  direction =  $\sigma_x$

$$\text{Therefore } \varepsilon_x = \frac{\sigma_x}{E}, \quad \varepsilon_y = -\mu \frac{\sigma_x}{E}, \quad \varepsilon_z = -\mu \frac{\sigma_x}{E}$$

IAS-5. Assuming  $E = 160$  GPa and  $G = 100$  GPa for a material, a strain tensor is given as: [IAS-2001]

$$\begin{pmatrix} 0.002 & 0.004 & 0.006 \\ 0.004 & 0.003 & 0 \\ 0.006 & 0 & 0 \end{pmatrix}$$

The shear stress,  $\tau_{xy}$  is:

(a) 400 MPa (b) 500 MPa (c) 800 MPa (d) 1000 MPa

IAS-5. Ans. (c)

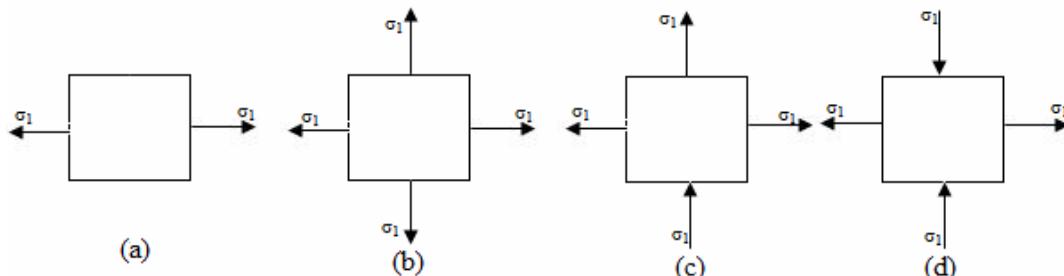
$$\begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix} \text{ and } \varepsilon_{xy} = \frac{\gamma_{xy}}{2}$$

$$\tau_{xy} = G \gamma_{xy} = 100 \times 10^3 \times (0.004 \times 2) \text{ MPa} = 800 \text{ MPa}$$

## Principal Stress and Principal Plane

IAS-6. A material element subjected to a plane state of stress such that the maximum shear stress is equal to the maximum tensile stress, would correspond to

[IAS-1998]



$$\text{IAS-6. Ans. (d)} \quad \tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{\sigma_1 - (-\sigma_1)}{2} = \sigma_1$$

IAS-7. A solid circular shaft is subjected to a maximum shearing stress of 140 MPa. The magnitude of the maximum normal stress developed in the shaft is:

## Chapter-2

## Principal Stress and Strain

(a) 140 MPa

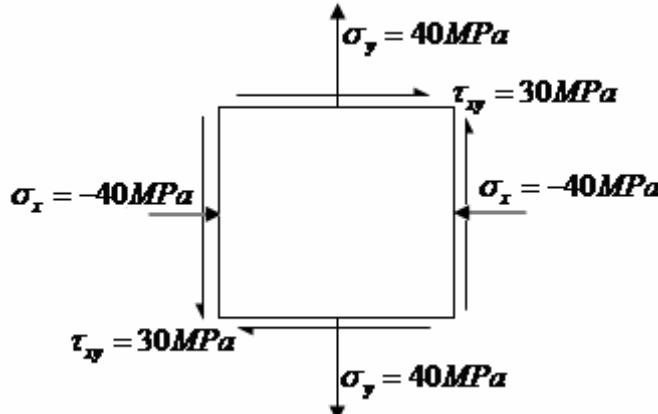
(b) 80 MPa

(c) 70 MPa

(d) 60 MPa

IAS-7. Ans. (a)  $\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2}$  Maximum normal stress will developed if  $\sigma_1 = -\sigma_2 = \sigma$

IAS-8. The state of stress at a point in a loaded member is shown in the figure. The magnitude of maximum shear stress is [1 MPa = 10 kg/cm<sup>2</sup>] [IAS 1994]  
 (a) 10 MPa (b) 30 MPa (c) 50 MPa (d) 100 MPa



IAS-8. Ans. (c)  $\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{-40 - 40}{2}\right)^2 + 30^2} = 50 \text{ MPa}$

IAS-9. A horizontal beam under bending has a maximum bending stress of 100 MPa and a maximum shear stress of 20 MPa. What is the maximum principal stress in the beam? [IAS-2004]

(a) 20

(b) 50

(c)  $50 + \sqrt{2900}$

(d) 100

IAS-9. Ans. (c)  $\sigma_b = 100 \text{ MPa}$   $\tau = 20 \text{ MPa}$

$$\sigma_{1,2} = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$\sigma_{1,2} = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2} = \frac{100}{2} + \sqrt{\left(\frac{100}{2}\right)^2 + 20^2} = (50 + \sqrt{2900}) \text{ MPa}$$

IAS-10. When the two principal stresses are equal and like: the resultant stress on any plane is: [IAS-2002]

(a) Equal to the principal stress

(b) Zero

(c) One half the principal stress

(d) One third of the principal stress

IAS-10. Ans. (a)  $\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta$

[We may consider this as  $\tau_{xy} = 0$ ]  $\sigma_x = \sigma_y = \sigma$  (say) So  $\sigma_n = \sigma$  for any plane

IAS-11. Assertion (A): When an isotropic, linearly elastic material is loaded biaxially, the directions of principal stressed are different from those of principal strains. [IAS-2001]

Reason (R): For an isotropic, linearly elastic material the Hooke's law gives only two independent material properties.

(a) Both A and R are individually true and R is the correct explanation of A

(b) Both A and R are individually true but R is NOT the correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

IAS-11. Ans. (d) They are same.

## Chapter-2

## Principal Stress and Strain

IAS-12. Principal stress at a point in a stressed solid are 400 MPa and 300 MPa respectively. The normal stresses on planes inclined at  $45^\circ$  to the principal planes will be: [IAS-2000]

(a) 200 MPa and 500 MPa (b) 350 MPa on both planes  
 (c) 100 MPa and 600 MPa (d) 150 MPa and 550 MPa

IAS-12. Ans. (b)

$$\sigma_n = \left( \frac{\sigma_x + \sigma_y}{2} \right) + \left( \frac{\sigma_x - \sigma_y}{2} \right) \cos 2\theta = \frac{400 + 300}{2} + \frac{400 - 300}{2} \cos 2 \times 45^\circ = 350 \text{ MPa}$$

IAS-13. The principal stresses at a point in an elastic material are 60 N/mm<sup>2</sup> tensile, 20 N/mm<sup>2</sup> tensile and 50 N/mm<sup>2</sup> compressive. If the material properties are:  $\mu = 0.35$  and  $E = 105 \text{ Nmm}^2$ , then the volumetric strain of the material is: [IAS-1997]

(a)  $9 \times 10^{-5}$  (b)  $3 \times 10^{-4}$  (c)  $10.5 \times 10^{-5}$  (d)  $21 \times 10^{-5}$

IAS-13. Ans. (a)

$$\epsilon_x = \frac{\sigma_x}{E} - \mu \left( \frac{\sigma_y}{E} + \frac{\sigma_z}{E} \right), \quad \epsilon_y = \frac{\sigma_y}{E} - \mu \left( \frac{\sigma_z}{E} + \frac{\sigma_x}{E} \right) \quad \text{and} \quad \epsilon_z = \frac{\sigma_z}{E} - \mu \left( \frac{\sigma_x}{E} + \frac{\sigma_y}{E} \right)$$

$$\begin{aligned} \epsilon_v &= \epsilon_x + \epsilon_y + \epsilon_z = \frac{\sigma_x + \sigma_y + \sigma_z}{E} - \frac{2\mu}{E} (\sigma_x + \sigma_y + \sigma_z) \\ &= (1 - 2\mu) \left( \frac{\sigma_x + \sigma_y + \sigma_z}{E} \right) = \left( \frac{60 + 20 - 50}{10^5} \right) (1 - 2 \times 0.35) = 9 \times 10^{-5} \end{aligned}$$

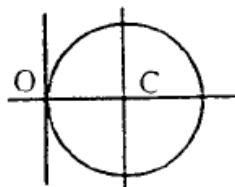
## Mohr's circle

IAS-14. Match List-I (Mohr's Circles of stress) with List-II (Types of Loading) and select the correct answer using the codes given below the lists: [IAS-2004]

List-I

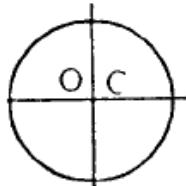
(Mohr's Circles of Stress)

A.



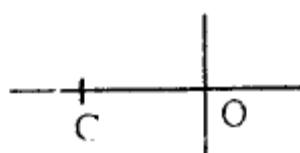
1. A shaft compressed all round by a hub

B.



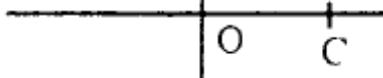
2. Bending moment applied at the free end of a cantilever

C.



3. Shaft under torsion

D.



4. Thin cylinder under pressure

5. Thin spherical shell under internal pressure

Codes:

A      B      C      D  
 (a)    5      4      3      2  
 (c)    4      3      2      5

A      B      C      D  
 (b)    2      4      1      3  
 (d)    2      3      1      5

IAS-14. Ans. (d)

## Chapter-2

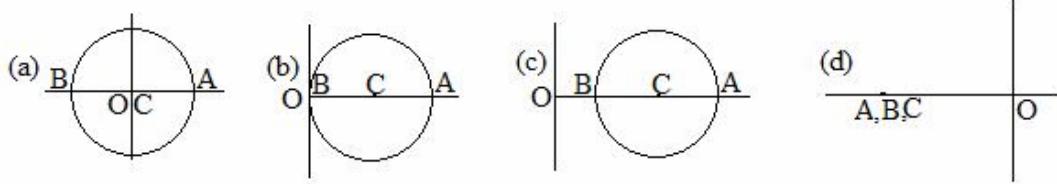
## Principal Stress and Strain

IAS-15. The resultant stress on a certain plane makes an angle of  $20^\circ$  with the normal to the plane. On the plane perpendicular to the above plane, the resultant stress makes an angle of  $\theta$  with the normal. The value of  $\theta$  can be: [IAS-2001]

(a)  $0^\circ$  or  $20^\circ$  (b) Any value other than  $0^\circ$  or  $90^\circ$   
 (c) Any value between  $0^\circ$  and  $20^\circ$  (d)  $20^\circ$  only

IAS-15. Ans. (b)

IAS-16. The correct Mohr's stress-circle drawn for a point in a solid shaft compressed by a shrunk fit hub is as (O-Origin and C-Centre of circle; OA =  $\sigma_1$  and OB =  $\sigma_2$ ) [IAS-2001]



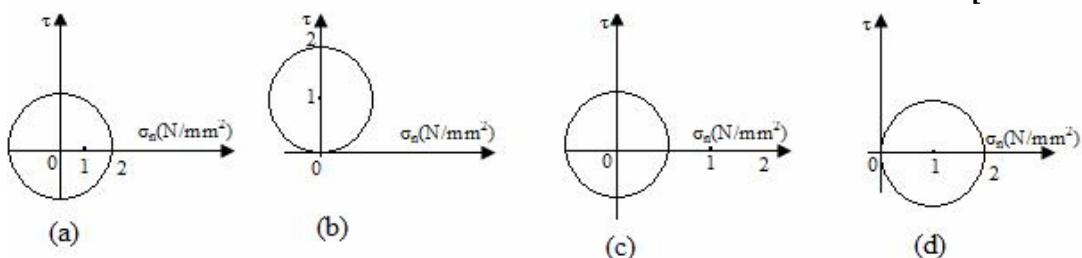
IAS-16. Ans. (d)

IAS-17. A Mohr's stress circle is drawn for a body subjected to tensile stress  $f_x$  and  $f_y$  in two mutually perpendicular directions such that  $f_x > f_y$ . Which one of the following statements in this regard is NOT correct? [IAS-2000]

(a) Normal stress on a plane at  $45^\circ$  to  $f_x$  is equal to  $\frac{f_x + f_y}{2}$   
 (b) Shear stress on a plane at  $45^\circ$  to  $f_x$  is equal to  $\frac{f_x - f_y}{2}$   
 (c) Maximum normal stress is equal to  $f_x$ .  
 (d) Maximum shear stress is equal to  $\frac{f_x + f_y}{2}$

IAS-17. Ans. (d) Maximum shear stress is  $\frac{f_x - f_y}{2}$

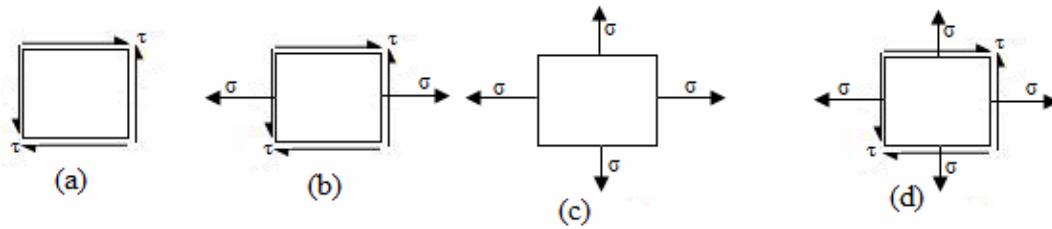
IAS-18. For the given stress condition  $\sigma_x = 2$  N/mm<sup>2</sup>,  $\sigma_y = 0$  and  $\tau_{xy} = 0$ , the correct Mohr's circle is: [IAS-1999]



IAS-18. Ans. (d) Centre  $\left(\frac{\sigma_x + \sigma_y}{2}, 0\right) = \left(\frac{2+0}{2}, 0\right) = (1, 0)$

$$\text{radius} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{2-0}{2}\right)^2 + 0} = 1$$

IAS-19. For which one of the following two-dimensional states of stress will the Mohr's stress circle degenerate into a point? [IAS-1996]



IAS-19. Ans. (c) Mohr's circle will be a point.

$$\text{Radius of the Mohr's circle} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \quad \therefore \tau_{xy} = 0 \text{ and } \sigma_x = \sigma_y = \sigma$$

## Principal strains

IAS-20. In an axi-symmetric plane strain problem, let  $u$  be the radial displacement at  $r$ . Then the strain components  $\varepsilon_r, \varepsilon_\theta, \gamma_{r\theta}$  are given by [IAS-1995]

$$(a) \varepsilon_r = \frac{u}{r}, \varepsilon_\theta = \frac{\partial u}{\partial r}, \gamma_{r\theta} = \frac{\partial^2 u}{\partial r \partial \theta} \quad (b) \varepsilon_r = \frac{\partial u}{\partial r}, \varepsilon_\theta = \frac{u}{r}, \gamma_{r\theta} = 0$$

$$(c) \varepsilon_r = \frac{u}{r}, \varepsilon_\theta = \frac{\partial u}{\partial r}, \gamma_{r\theta} = 0 \quad (d) \varepsilon_r = \frac{\partial u}{\partial r}, \varepsilon_\theta = \frac{\partial u}{\partial \theta}, \gamma_{r\theta} = \frac{\partial^2 u}{\partial r \partial \theta}$$

IAS-20. Ans. (b)

IAS-21. Assertion (A): Uniaxial stress normally gives rise to triaxial strain.

Reason (R): Magnitude of strains in the perpendicular directions of applied stress is smaller than that in the direction of applied stress. [IAS-2004]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-21. Ans. (b)

IAS-22. Assertion (A): A plane state of stress will, in general, not result in a plane state of strain. [IAS-2002]

Reason (R): A thin plane lamina stretched in its own plane will result in a state of plane strain.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IAS-22. Ans. (c) R is false. Stress in one plane always induce a lateral strain with its orthogonal plane.

## Previous Conventional Questions with Answers

### Conventional Question IES-1999

**Question:** What are principal in planes?

**Answer:** The planes which pass through the point in such a manner that the resultant stress across them is totally a normal stress are known as principal planes. No shear stress exists at the principal planes.

### Conventional Question IES-2009

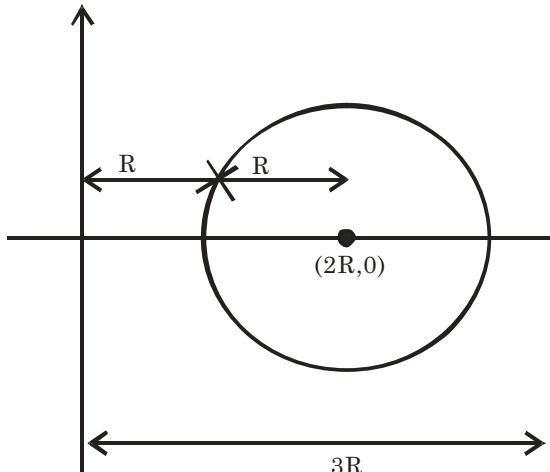
**Q.** The Mohr's circle for a plane stress is a circle of radius R with its origin at  $+2R$  on  $\sigma$  axis. Sketch the Mohr's circle and determine  $\sigma_{\max}$ ,  $\sigma_{\min}$ ,  $\sigma_{av}$ ,  $(\tau_{xy})_{\max}$  for this situation. [2 Marks]

**Ans.** Here  $\sigma_{\max} = 3R$

$$\sigma_{\min} = R$$

$$\sigma_{av} = \frac{3R + R}{2} = 2R$$

$$\text{and } \tau_{xy} = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{3R - R}{2} = R$$



### Conventional Question IES-1999

**Question:** Direct tensile stresses of 120 MPa and 70 MPa act on a body on mutually perpendicular planes. What is the magnitude of shearing stress that can be applied so that the major principal stress at the point does not exceed 135 MPa? Determine the value of minor principal stress and the maximum shear stress.

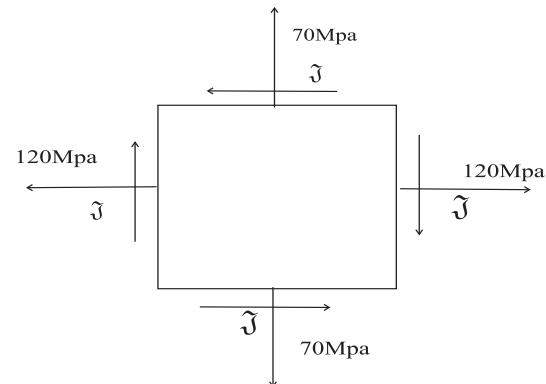
**Answer:** Let shearing stress is ' $\tau$ ' MPa.

The principal stresses are

$$\sigma_{1,2} = \frac{120 + 70}{2} \pm \sqrt{\left(\frac{120 - 70}{2}\right)^2 + \tau^2}$$

Major principal stress is

$$\begin{aligned} \sigma_1 &= \frac{120 + 70}{2} + \sqrt{\left(\frac{120 - 70}{2}\right)^2 + \tau^2} \\ &= 135 \text{ (Given) or, } \tau = 31.2 \text{ MPa.} \end{aligned}$$



Minor principal stress is

$$\sigma_2 = \frac{120 + 70}{2} - \sqrt{\left(\frac{120 - 70}{2}\right)^2 + 31.2^2} = 55 \text{ MPa}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{135 - 55}{2} = 40 \text{ MPa}$$

### Conventional Question IES-2009

Q. The state of stress at a point in a loaded machine member is given by the principle stresses. [ 2 Marks]

$$\sigma_1 = 600 \text{ MPa}, \sigma_2 = 0 \text{ and } \sigma_3 = -600 \text{ MPa}.$$

- What is the magnitude of the maximum shear stress?
- What is the inclination of the plane on which the maximum shear stress acts with respect to the plane on which the maximum principle stress  $\sigma_1$  acts?

Ans. (i) Maximum shear stress,

$$\tau = \frac{\sigma_1 - \sigma_3}{2} = \frac{600 - (-600)}{2} = 600 \text{ MPa}$$

(ii) At  $\theta = 45^\circ$  max. shear stress occurs with  $\sigma_1$  plane. Since  $\sigma_1$  and  $\sigma_3$  are principle stress does not contain shear stress. Hence max. shear stress is at  $45^\circ$  with principle plane.

### Conventional Question IES-2008

Question: A prismatic bar in compression has a cross- sectional area  $A = 900 \text{ mm}^2$  and carries an axial load  $P = 90 \text{ kN}$ . What are the stresses acts on

- A plane transverse to the loading axis;
- A plane at  $\theta = 60^\circ$  to the loading axis?

Answer: (i) From figure it is clear A plane transverse to loading axis,  $\theta = 0^\circ$

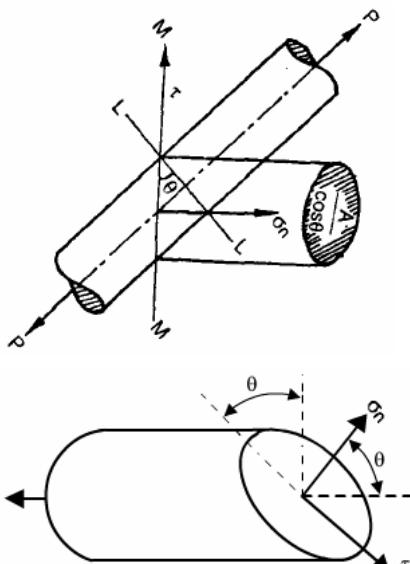
$$\therefore \sigma_n = \frac{P}{A} \cos^2 \theta = \frac{90000}{900} N / mm^2 = 100 N / mm^2$$

$$\text{and } \tau = \frac{P}{2A} \sin 2\theta = \frac{90000}{2 \times 900} \times \sin \theta = 0$$

(iii) A plane at  $60^\circ$  to loading axis,  $\theta = 60^\circ - 30^\circ = 30^\circ$

$$\sigma_n = \frac{P}{A} \cos^2 \theta = \frac{90000}{900} \times \cos^2 30^\circ = 75 N / mm^2$$

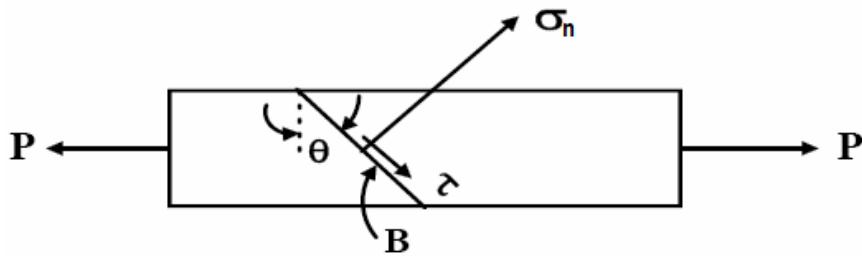
$$\tau = \frac{P}{2A} \sin 2\theta = \frac{90000}{2 \times 900} \sin 2 \times 60^\circ = 43.3 N / mm^2$$



### Conventional Question IES-2001

Question: A tension member with a cross-sectional area of  $30 \text{ mm}^2$  resists a load of  $80 \text{ kN}$ , Calculate the normal and shear stresses on the plane of maximum shear stress.

$$\sigma_n = \frac{P}{A} \cos^2 \theta \quad \tau = \frac{P}{2A} \sin 2\theta$$



For maximum shear stress  $\sin 2\theta = 1$ , or,  $\theta = 45^\circ$

$$(\sigma_n) = \frac{80 \times 10^3}{30} \times \cos^2 45 = 1333 \text{ MPa} \text{ and } \tau_{\max} = \frac{P}{2A} = \frac{80 \times 10^3}{30 \times 2} = 1333 \text{ MPa}$$

### Conventional Question IES-2007

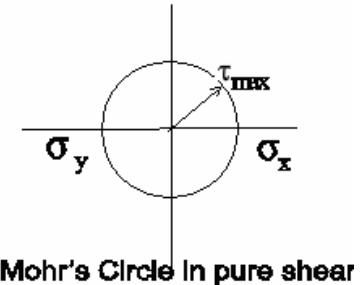
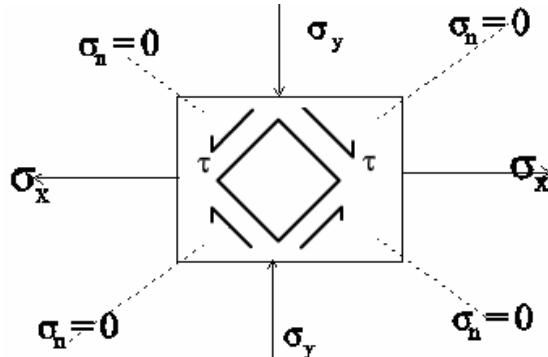
**Question:** At a point in a loaded structure, a pure shear stress state  $\tau = \pm 400 \text{ MPa}$  prevails on two given planes at right angles.

(i) What would be the state of stress across the planes of an element taken at  $+45^\circ$  to the given planes?

(ii) What are the magnitudes of these stresses?

**Answer:** (i) For pure shear

$$\sigma_x = -\sigma_y; \quad \tau_{\max} = \pm \sigma_x = \pm 400 \text{ MPa}$$



(ii) Magnitude of these stresses

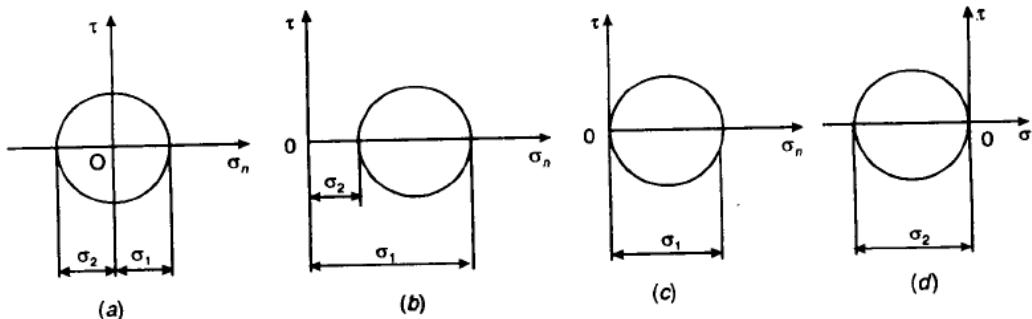
$$\sigma_n = \tau_{xy} \sin 2\theta = \tau_{xy} \sin 90^\circ = \tau_{xy} = 400 \text{ MPa} \text{ and } \tau = (-\tau_{xy} \cos 2\theta) = 0$$

### Conventional Question IAS-1997

**Question:** Draw Mohr's circle for a 2-dimensional stress field subjected to

(a) Pure shear (b) Pure biaxial tension (c) Pure uniaxial tension and (d) Pure uniaxial compression

**Answer:** Mohr's circles for 2-dimensional stress field subjected to pure shear, pure biaxial tension, pure uniaxial compression and pure uniaxial tension are shown in figure below:

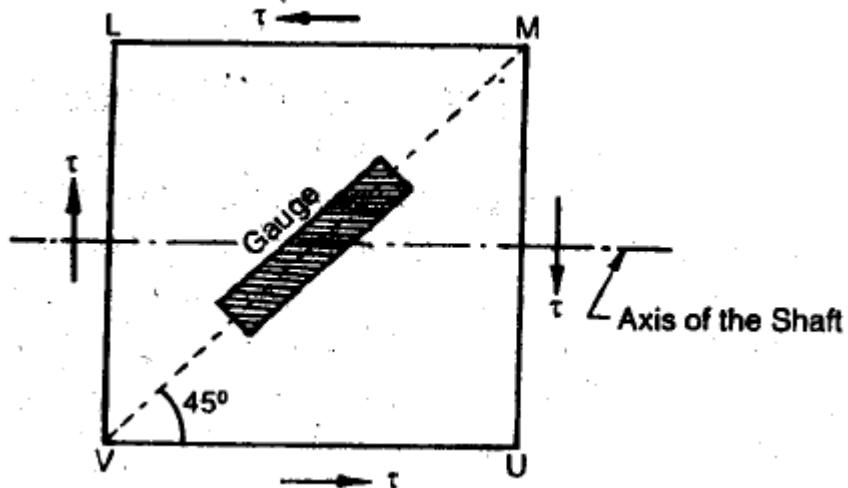


### Conventional Question IES-2003

**Question:** A Solid phosphor bronze shaft Page 105 of 429 in diameter is rotating at 800 rpm and transmitting power. It is subjected to torsion only. An electrical resistance

strain gauge mounted on the surface of the shaft with its axis at  $45^\circ$  to the shaft axis, gives the strain reading as  $3.98 \times 10^{-4}$ . If the modulus of elasticity for bronze is  $105 \text{ GN/m}^2$  and Poisson's ratio is 0.3, find the power being transmitted by the shaft. Bending effect may be neglected.

**Answer:**



Let us assume maximum shear stress on the cross-sectional plane MU is  $\tau$ . Then

$$\text{Principal stress along, } VM = -\frac{1}{2}\sqrt{4\tau^2} = -\tau \text{ (compressive)}$$

$$\text{Principal stress along, } LU = \frac{1}{2}\sqrt{4\tau^2} = \tau \text{ (tensile)}$$

Thus magnitude of the compressive strain along VM is

$$=\frac{\tau}{E}(1+\mu)=3.98 \times 10^{-4}$$

$$\text{or } \tau=\frac{3.98 \times 10^{-4} \times (105 \times 10^9)}{(1+0.3)}=32.15 \text{ MPa}$$

$$\therefore \text{Torque being transmitted (T)} = \tau \times \frac{\pi}{16} \times d^3$$

$$=(32.15 \times 10^6) \times \frac{\pi}{16} \times 0.06^3 = 1363.5 \text{ Nm}$$

$$\therefore \text{Power being transmitted, } P = T \cdot \omega = T \cdot \left( \frac{2\pi N}{60} \right) = 1363.5 \times \left( \frac{2\pi \times 800}{60} \right) W = 114.23 \text{ kW}$$

### Conventional Question IES-2002

**Question:** The magnitude of normal stress on two mutually perpendicular planes, at a point in an elastic body are 60 MPa (compressive) and 80 MPa (tensile) respectively. Find the magnitudes of shearing stresses on these planes if the magnitude of one of the principal stresses is 100 MPa (tensile). Find also the magnitude of the other principal stress at this point.

## Chapter-2

## Principal Stress and Strain

**Answer:**

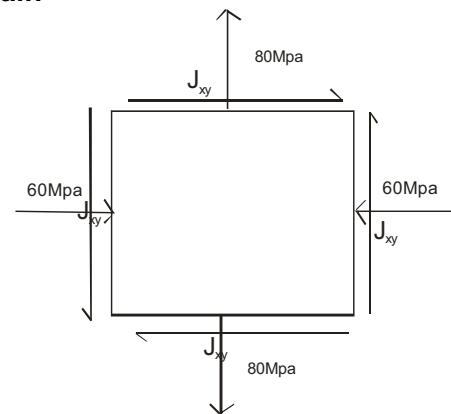
Above figure shows stress condition assuming shear stress is ' $\tau_{xy}$ '

Principal stresses

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\text{or, } \sigma_{1,2} = \frac{-60 + 80}{2} \pm \sqrt{\left(\frac{-60 - 80}{2}\right)^2 + \tau_{xy}^2}$$

$$\text{or, } \sigma_{1,2} = \frac{-60 + 80}{2} \pm \sqrt{\left(\frac{-60 - 80}{2}\right)^2 + \tau_{xy}^2}$$



To make principal stress 100 MPa we have to consider '+'.

$$\therefore \sigma_1 = 100 \text{ MPa} = 10 + \sqrt{70^2 + \tau_{xy}^2}; \text{ or, } \tau_{xy} = 56.57 \text{ MPa}$$

Therefore other principal stress will be

$$\sigma_2 = \frac{-60 + 80}{2} - \sqrt{\left(\frac{-60 - 80}{2}\right)^2 + (56.57)^2}$$

i.e. 80 MPa(compressive)

### Conventional Question IES-2001

**Question:** A steel tube of inner diameter 100 mm and wall thickness 5 mm is subjected to a torsional moment of 1000 Nm. Calculate the principal stresses and orientations of the principal planes on the outer surface of the tube.

**Answer:**

$$\text{Polar moment of Inertia (J)} = \frac{\pi}{32} \left[ (0.110)^4 - (0.100)^4 \right] = 4.56 \times 10^{-6} \text{ m}^4$$

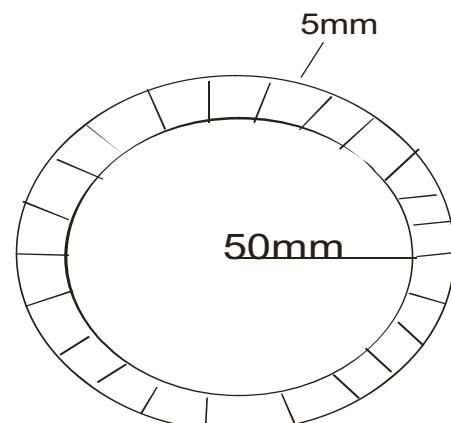
$$\text{Now } \frac{T}{J} = \frac{\tau}{R} \text{ or } J = \frac{T \cdot R}{J} = \frac{1000 \times (0.055)}{4.56 \times 10^{-6}} = 12.07 \text{ MPa}$$

$$\text{Now, } \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \infty,$$

gives  $\theta_p = 45^\circ$  or  $135^\circ$

$$\therefore \sigma_1 = \tau_{xy} \sin 2\theta = 12.07 \times \sin 90^\circ = 12.07 \text{ MPa}$$

$$\text{and } \sigma_2 = 12.07 \sin 270^\circ = -12.07 \text{ MPa}$$



### Conventional Question IES-2000

**Question:** At a point in a two dimensional stress system the normal stresses on two mutually perpendicular planes are  $\sigma_x$  and  $\sigma_y$  and the shear stress is  $\tau_{xy}$ . At what value of shear stress, one of the principal stresses will become zero?

**Answer:**

Two principal stressdes are

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Considering (-)ive sign it may be zero

$$\therefore \left( \frac{\sigma_x + \sigma_y}{2} \right) = \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2} \quad \text{or,} \quad \left( \frac{\sigma_x + \sigma_y}{2} \right)^2 = \left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2$$

$$\text{or,} \quad \tau_{xy}^2 = \left( \frac{\sigma_x + \sigma_y}{2} \right)^2 - \left( \frac{\sigma_x - \sigma_y}{2} \right)^2 \quad \text{or,} \quad \tau_{xy}^2 = \sigma_x \sigma_y \quad \text{or,} \quad \tau_{xy} = \pm \sqrt{\sigma_x \sigma_y}$$

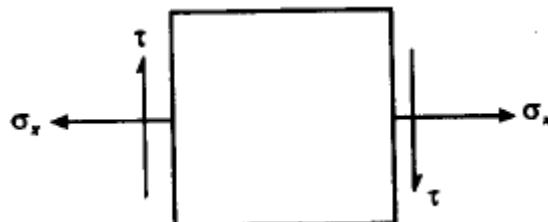
### Conventional Question IES-1996

**Question:** A solid shaft of diameter 30 mm is fixed at one end. It is subject to a tensile force of 10 kN and a torque of 60 Nm. At a point on the surface of the shaft, determine the principle stresses and the maximum shear stress.

**Answer:** Given: D = 30 mm = 0.03 m; P = 10 kN; T = 60 Nm

Principal stresses ( $\sigma_1, \sigma_2$ ) and maximum shear stress ( $\tau_{\max}$ ):

$$\text{Tensile stress } \sigma_t = \sigma_x = \frac{10 \times 10^3}{\frac{\pi}{4} \times 0.03^2} = 14.15 \times 10^6 \text{ N/m}^2 \text{ or } 14.15 \text{ MN/m}^2$$



$$\text{As per torsion equation, } \frac{T}{J} = \frac{\tau}{R}$$

$$\therefore \text{Shear stress, } \tau = \frac{TR}{J} = \frac{TR}{\frac{\pi}{32} D^4} = \frac{60 \times 0.015}{\frac{\pi}{32} \times (0.03)^4} = 11.32 \times 10^6 \text{ N/m}^2$$

or  $11.32 \text{ MN/m}^2$

The principal stresses are calculated by using the relations:

$$\sigma_{1,2} = \left( \frac{\sigma_x + \sigma_y}{2} \right) \pm \sqrt{\left( \frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2}$$

$$\text{Here } \sigma_x = 14.15 \text{ MN/m}^2, \sigma_y = 0; \tau_{xy} = \tau = 11.32 \text{ MN/m}^2$$

$$\therefore \sigma_{1,2} = \frac{14.15}{2} \pm \sqrt{\left( \frac{14.15}{2} \right)^2 + (11.32)^2}$$

$$= 7.07 \pm 13.35 = 20.425 \text{ MN/m}^2, -6.275 \text{ MN/m}^2.$$

Hence, major principal stress,  $\sigma_1 = 20.425 \text{ MN/m}^2$  (tensile)

Minor principal stress,  $\sigma_2 = 6.275 \text{ MN/m}^2$  (compressive)

$$\text{Maximum shear stress, } \tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{20.425 - (-6.275)}{2} = 13.35 \text{ MN/m}^2$$

### Conventional Question IES-2000

**Question:** Two planes AB and BC which are at right angles are acted upon by tensile stress of  $140 \text{ N/mm}^2$  and a compressive stress of  $70 \text{ N/mm}^2$  respectively and also by stress  $35 \text{ N/mm}^2$ . Determine the principal stresses and principal planes. Find also the maximum shear stress and planes on which they act.

Sketch the Mohr circle and mark the relevant data.

## Chapter-2

## Principal Stress and Strain

Answer:

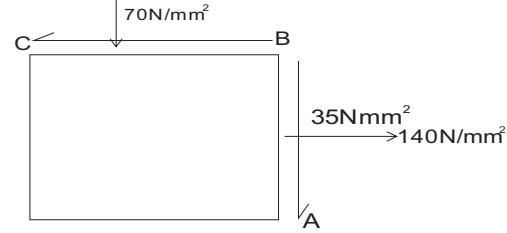
Given

$$\sigma_x = 140 \text{ MPa (tensile)}$$

$$\sigma_y = -70 \text{ MPa (compressive)}$$

$$\tau_{xy} = 35 \text{ MPa}$$

Principal stresses;  $\sigma_1, \sigma_2$ :



$$\text{We know that, } \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{140 - 70}{2} \pm \sqrt{\left(\frac{140 + 70}{2}\right)^2 + 35^2} = 35 \pm 110.7$$

Therefore  $\sigma_1 = 145.7 \text{ MPa}$  and  $\sigma_2 = -75.7 \text{ MPa}$

Position of Principal planes  $\theta_1, \theta_2$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 35}{140 + 70} = 0.3333$$

$$\text{Maximum shear stress, } \tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{145 + 75.7}{2} = 110.7 \text{ MPa}$$

Mohr circle:

$$OL = \sigma_x = 140 \text{ MPa}$$

$$OM = \sigma_y = -70 \text{ MPa}$$

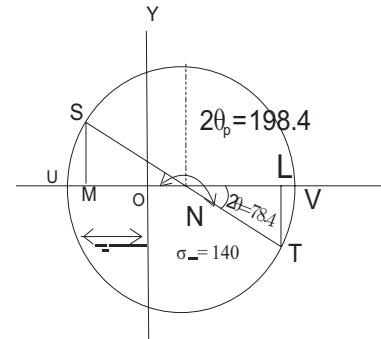
$$SM = LT = \tau_{xy} = 35 \text{ MPa}$$

Joining ST that cuts at 'N'

$$SN = NT = \text{radius of Mohr circle} = 110.7 \text{ MPa}$$

$$OV = \sigma_1 = 145.7 \text{ MPa}$$

$$OV = \sigma_2 = -75.7 \text{ MPa}$$



## Conventional Question IES-2010

Q6. The data obtained from a rectangular strain gauge rosette attached to a stressed steel member are  $\epsilon_0 = -220 \times 10^{-6}$ ,  $\epsilon_{45}^0 = 120 \times 10^{-6}$ , and  $\epsilon_{90} = 220 \times 10^{-6}$ . Given that the value of  $E = 2 \times 10^5 \text{ N/mm}^2$  and Poisson's Ratio  $\mu = 0.3$ , calculate the values of principal stresses acting at the point and their directions. [10 Marks]

Ans. A rectangular strain gauge rosette strain

$$\epsilon_0 = -220 \times 10^{-6} \quad \epsilon_{45}^0 = 120 \times 10^{-6} \quad \epsilon_{90} = 220 \times 10^{-6}$$

$$E = 2 \times 10^{11} \text{ N/m}^2 \quad \text{poisson ratio } \mu = 0.3$$

Find out principal stress and their direction.

Let  $e_a = \epsilon_0$   $e_c = \epsilon_{90}$  and  $e_b = \epsilon_{45}$

We know that principal strain are

$$\epsilon_{12} = \frac{e_a + e_b}{2} \sqrt{(e_a - e_b)^2 + (e_b - e_c)^2}$$

$$\Rightarrow \frac{(-220 \times 10^{-6} + 120 \times 10^{-6})}{2} \pm \frac{1}{\sqrt{2}} \sqrt{((-220 - 120) \times 10^{-6})^2 + ((120 - 220) \times 10^{-6})^2}$$

$$\Rightarrow -50 \times 10^{-6} \pm \frac{1}{\sqrt{2}} 354.40 \times 10^{-6}$$

$$\epsilon_{12} \Rightarrow -50 \times 10^{-6} \pm 250.6 \times 10^{-6}$$

$$\epsilon_1 = 2.01 \times 10^{-4}$$

$$\epsilon_2 = -3.01 \times 10^{-4}$$

Direction can be find out :-

$$\tan 2\theta_{pe} = \frac{2e_b - e_a - e_c}{e_c - e_a} = \frac{2 \times 120 \times 10^{-6}}{220 \times 10^{-6} + 220 \times 10^{-6}}$$

$$\Rightarrow \frac{240}{440} = 0.55$$

$$2\theta_{pe} = 28.81$$

$$\theta_{pe} = 14.45^\circ \text{ clockwise from principal strain } t_1$$

Principal stress:-

$$\sigma_1 \frac{E(\epsilon_1 + \mu \epsilon_2)}{1 - \mu^2} = \frac{2 \times 10^{11} (2 + 0.3(-3) \times 10^{-4})}{1 - 0.3^2}$$

$$= 241.78 \times 10^5 \text{ N/m}^2$$

$$= -527.47 \times 10^5 \text{ N/m}^2$$

### Conventional Question IES-1998

**Question:** When using strain-gauge system for stress/force/displacement measurements how are in-built magnification and temperature compensation achieved?

**Answer:** In-built magnification and temperature compensation are achieved by

- Through use of adjacent arm balancing of Wheat-stone bridge.
- By means of self temperature compensation by selected melt-gauge and dual element-gauge.

### Conventional Question AMIE-1998

**Question:** A cylinder (500 mm internal diameter and 20 mm wall thickness) with closed ends is subjected simultaneously to an internal pressure of 0.60 MPa, bending moment 64000 Nm and torque 16000 Nm. Determine the maximum tensile stress and shearing stress in the wall.

**Answer:** Given:  $d = 500 \text{ mm} = 0.5 \text{ m}$ ;  $t = 20 \text{ mm} = 0.02 \text{ m}$ ;  $p = 0.60 \text{ MPa} = 0.6 \text{ MN/m}^2$ ;  $M = 64000 \text{ Nm} = 0.064 \text{ MNm}$ ;  $T = 16000 \text{ Nm} = 0.016 \text{ MNm}$ .

Maximum tensile stress:

First let us determine the principle stresses  $\sigma_1$  and  $\sigma_2$  assuming this as a thin cylinder.

$$\text{We know, } \sigma_1 = \frac{pd}{2t} = \frac{0.6 \times 0.5}{2 \times 0.02} = 7.5 \text{ MN/m}^2$$

$$\text{and } \sigma_2 = \frac{pd}{4t} = \frac{0.6 \times 0.5}{4 \times 0.02} = 3.75 \text{ MN/m}^2$$

Next consider effect of combined bending moment and torque on the walls of the cylinder. Then the principal stresses  $\sigma'_1$  and  $\sigma'_2$  are given by

**Chapter-2****Principal Stress and Strain**

$$\sigma'_1 = \frac{16}{\pi d^3} \left[ M + \sqrt{M^2 + T^2} \right]$$

$$\text{and } \sigma'_2 = \frac{16}{\pi d^3} \left[ M - \sqrt{M^2 + T^2} \right]$$

$$\therefore \sigma'_1 = \frac{16}{\pi \times (0.5)^3} \left[ 0.064 + \sqrt{0.064^2 + 0.016^2} \right] = 5.29 \text{ MN/m}^2$$

$$\text{and } \sigma'_2 = \frac{16}{\pi \times (0.5)^3} \left[ 0.064 - \sqrt{0.064^2 + 0.016^2} \right] = -0.08 \text{ MN/m}^2$$

Maximum shearing stress,  $\tau_{\max}$  :

$$\text{We Know, } \tau_{\max} = \frac{\sigma_I - \sigma_{II}}{2}$$

$$\sigma_{II} = \sigma_2 + \sigma'_2 = 3.75 - 0.08 = 3.67 \text{ MN/m}^2 \text{ (tensile)}$$

$$\therefore \tau_{\max} = \frac{12.79 - 3.67}{2} = 4.56 \text{ MN/m}^2$$

### 3.

## Moment of Inertia and Centroid

### Theory at a Glance (for IES, GATE, PSU)

#### 3.1 Centre of gravity

The centre of gravity of a body defined as the point through which the whole weight of a body may be assumed to act.

#### 3.2 Centroid or Centre of area

The centroid or centre of area is defined as the point where the whole area of the figure is assumed to be concentrated.

#### 3.3 Moment of Inertia (MOI)

- About any point the product of the force and the perpendicular distance between them is known as moment of a force or first moment of force.
- This first moment is again multiplied by the perpendicular distance between them to obtain second moment of force.
- In the same way if we consider the area of the figure it is called second moment of area or area moment of inertia and if we consider the mass of a body it is called second moment of mass or mass moment of Inertia.
- ***Mass moment of inertia*** is the measure of resistance of the body to rotation and ***forms the basis of dynamics of rigid bodies.***
- ***Area moment of Inertia*** is the measure of resistance to bending and ***forms the basis of strength of materials.***

#### 3.4 Mass moment of Inertia (MOI)

$$I = \sum_i m_i r_i^2$$

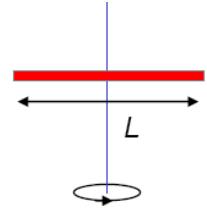
- Notice that the moment of inertia 'I' depends on the distribution of mass in the system.
- The furthest the mass is from the rotation axis, the bigger the moment of inertia.
- For a given object, the moment of inertia depends on where we choose the rotation axis.
- In rotational dynamics, the moment of inertia 'I' appears in the same way that mass  $m$  does in linear dynamics.

### Chapter-3

### Moment of Inertia and Centroid

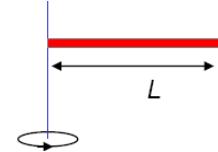
- *Solid disc or cylinder of mass M and radius R*, about perpendicular axis through its centre,  $I = \frac{1}{2}MR^2$
- Solid sphere of mass  $M$  and radius  $R$ , about an axis through its centre,  $I = 2/5 M R^2$
- Thin *rod of mass M and length L*, about a perpendicular axis through its centre.

$$I = \frac{1}{12}ML^2$$



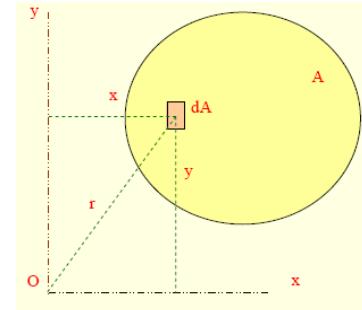
- Thin rod of mass  $M$  and length  $L$ , about a perpendicular axis through its end.

$$I = \frac{1}{3}ML^2$$



### 3.5 Area Moment of Inertia (MOI) or Second moment of area

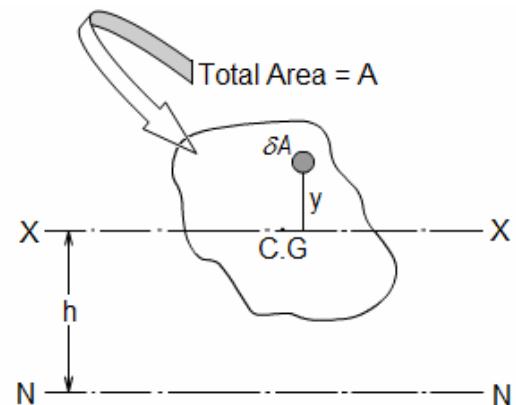
- To find the centroid of an area by the first moment of the area about an axis was determined ( $\int x dA$ )
- Integral of the *second moment of area* is called moment of inertia ( $\int x^2 dA$ )
- Consider the area ( $A$ )
- By definition, the moment of inertia of the differential area about the x and y axes are  $dI_{xx}$  and  $dI_{yy}$
- $dI_{xx} = y^2 dA$        $I_{xx} = \int y^2 dA$
- $dI_{yy} = x^2 dA$        $I_{yy} = \int x^2 dA$



### 3.6 Parallel axis theorem for an area

The rotational inertia about any axis is the sum of second moment of inertia about a parallel axis through the C.G and total area of the body times square of the distance between the axes.

$$I_{NN} = I_{CG} + Ah^2$$

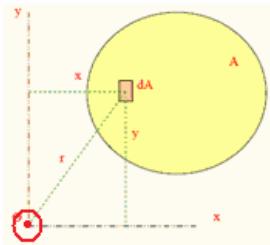


### 3.7 Perpendicular axis theorem for an area

If  $x$ ,  $y$  &  $z$  are mutually perpendicular axes as shown, then

$$I_{zz}(J) = I_{xx} + I_{yy}$$

Z-axis is perpendicular to the plane of  $x - y$  and vertical to this page as shown in figure.



- To find the moment of inertia of the differential area about the pole (point of origin) or z-axis,  $(r)$  is used.  $(r)$  is the perpendicular distance from the pole to  $dA$  for the entire area

$$J = \int r^2 dA = \int (x^2 + y^2) dA = I_{xx} + I_{yy} \text{ (since } r^2 = x^2 + y^2\text{)}$$

Where,  $J$  = polar moment of inertia

### 3.8 Moments of Inertia (area) of some common area

#### (i) MOI of Rectangular area

Moment of inertia about axis XX which passes through centroid.

Take an element of width 'dy' at a distance  $y$  from XX axis.

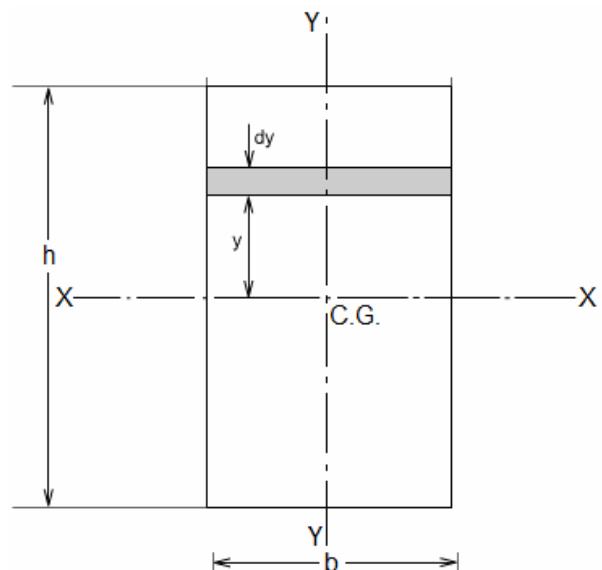
$\therefore$  Area of the element  $(dA) = b \times dy$ .

and Moment of Inertia of the element about XX

$$\text{axis} = dA \times y^2 = b \cdot y^2 \cdot dy$$

$\therefore$  Total MOI about XX axis (Note it is area moment of Inertia)

$$I_{xx} = \int_{-h/2}^{+h/2} b y^2 dy = 2 \int_0^{h/2} b y^2 dy = \frac{bh^3}{12}$$



$$I_{xx} = \frac{bh^3}{12}$$

$$\text{Similarly, we may find, } I_{yy} = \frac{hb^3}{12}$$

$$\therefore \text{Polar moment of inertia (J)} = I_{xx} + I_{yy} = \frac{bh^3}{12} + \frac{hb^3}{12}$$

### Chapter-3

### Moment of Inertia and Centroid

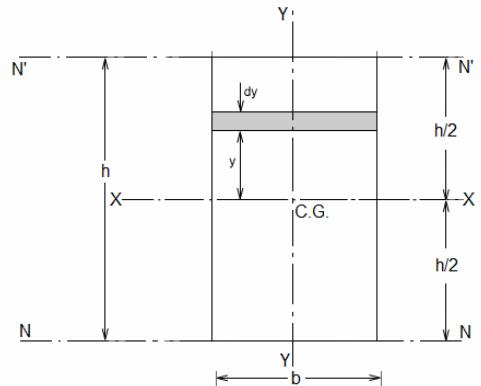
If we want to know the **MOI about an axis NN** passing

through the bottom edge or top edge.

Axis XX and NN are parallel and at a distance  $h/2$ .

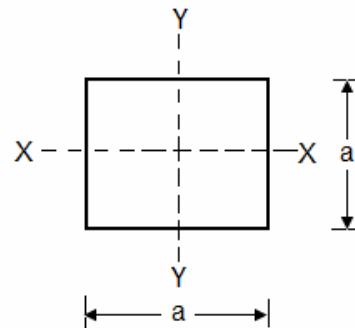
Therefore  $I_{NN} = I_{xx} + \text{Area} \times (\text{distance})^2$

$$= \frac{bh^3}{12} + b \times h \times \left(\frac{h}{2}\right)^2 = \frac{bh^3}{3}$$



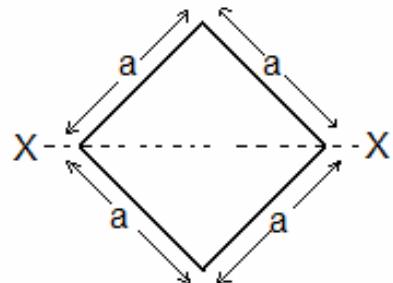
### Case-I: Square area

$$I_{xx} = \frac{a^4}{12}$$



### Case-II: Square area with diagonal as axis

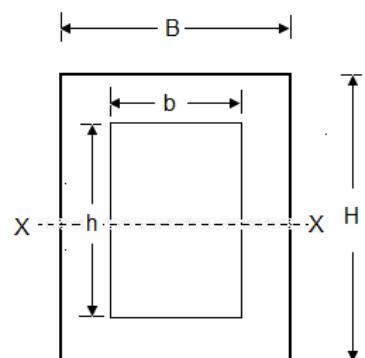
$$I_{xx} = \frac{a^4}{12}$$



### Case-III: Rectangular area with a centrally rectangular hole

Moment of inertia of the area = moment of inertia of BIG rectangle – moment of inertia of SMALL rectangle

$$I_{xx} = \frac{BH^3}{12} - \frac{bh^3}{12}$$



### Chapter-3

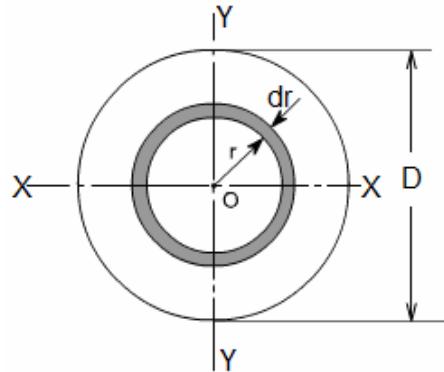
### Moment of Inertia and Centroid

#### (ii) MOI of a Circular area

The moment of inertia about axis XX this passes through the centroid. It is very easy to find polar moment of inertia about point 'O'. Take an element of width 'dr' at a distance 'r' from centre. Therefore, the moment of inertia of this element about polar axis

$$d(J) = d(I_{xx} + I_{yy}) = \text{area of ring} \times (\text{radius})^2$$

or  $d(J) = 2\pi r dr \times r^2$



Integrating both side we get

$$J = \int_0^R 2\pi r^3 dr = \frac{\pi R^4}{2} = \frac{\pi D^4}{32}$$

Due to symmetry  $I_{xx} = I_{yy}$

$$\text{Therefore, } I_{xx} = I_{yy} = \frac{J}{2} = \frac{\pi D^4}{64}$$

$$I_{xx} = I_{yy} = \frac{\pi D^4}{64} \quad \text{and} \quad J = \frac{\pi D^4}{32}$$

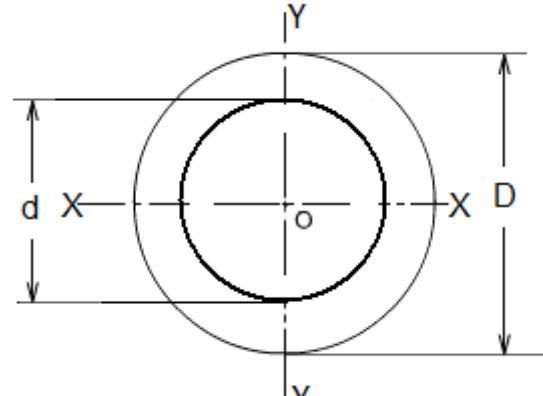
#### Case-I: Moment of inertia of a circular area with a concentric hole.

Moment of inertia of the area = moment of inertia of BIG circle – moment of inertia of SMALL circle.

$$I_{xx} = I_{yy} = \frac{\pi D^4}{64} - \frac{\pi d^4}{64}$$

$$= \frac{\pi}{64}(D^4 - d^4)$$

$$\text{and } J = \frac{\pi}{32}(D^4 - d^4)$$



#### Case-II: Moment of inertia of a semi-circular area.

$I_{NN} = \frac{1}{2}$  of the moment of total circular lamina

$$= \frac{1}{2} \times \left( \frac{\pi D^4}{64} \right) = \frac{\pi D^4}{128}$$

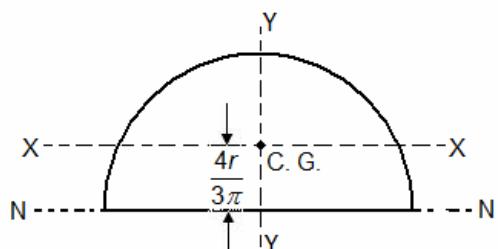


We know that distance of CG from base is

$$\frac{4r}{3\pi} = \frac{2D}{3\pi} = h \text{ (say)}$$

i.e. distance of parallel axis XX and NN is (h)

∴ According to parallel axis theory



$$I_{NN} = I_G + \text{Area} \times (\text{distance})^2$$

$$\text{or } \frac{\pi D^4}{128} = I_{xx} + \frac{1}{2} \left( \frac{\pi D^2}{4} \right) \times (h)^2$$

$$\text{or } \frac{\pi D^4}{128} = I_{xx} + \frac{1}{2} \times \left( \frac{\pi D^2}{4} \right) \times \left( \frac{2D}{3\pi} \right)$$

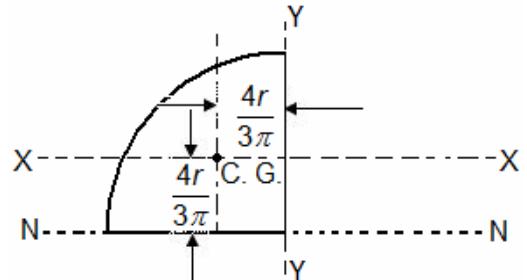
or  $I_{xx} = 0.11R^4$

### Case - III: Quarter circle area

$I_{xx}$  = one half of the moment of Inertia of the Semi-circular area about XX.

$$I_{xx} = \frac{1}{2} \times (0.11R^4) = 0.055 R^4$$

$I_{xx} = 0.055 R^4$



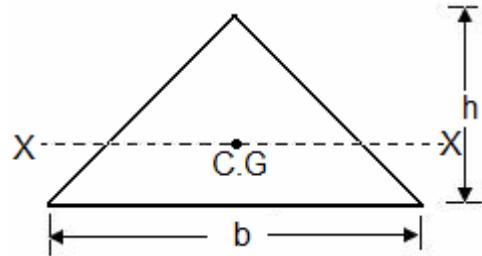
$I_{NN}$  = one half of the moment of Inertia of the Semi-circular area about NN.

$$\therefore I_{NN} = \frac{1}{2} \times \frac{\pi D^4}{64} = \frac{\pi D^4}{128}$$

### (iii) Moment of Inertia of a Triangular area

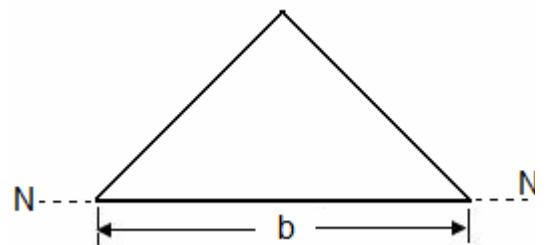
(a) Moment of Inertia of a Triangular area of a axis XX parallel to base and passes through C.G.

$$I_{xx} = \frac{bh^3}{36}$$



(b) Moment of inertia of a triangle about an axis passes through base

$$I_{NN} = \frac{bh^3}{12}$$



### Chapter-3

### Moment of Inertia and Centroid

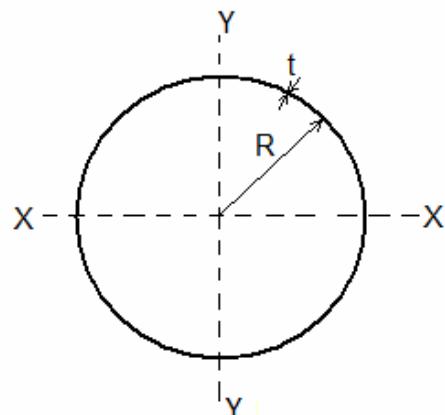
#### (iv) Moment of inertia of a thin circular ring:

##### Polar moment of Inertia

$$(J) = R^2 \times \text{area of whole ring}$$

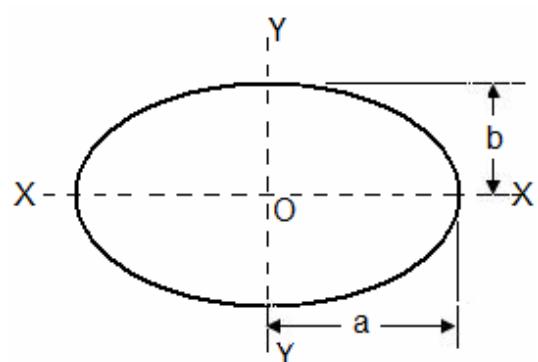
$$= R^2 \times 2\pi R t = 2\pi R^3 t$$

$$I_{xx} = I_{yy} = \frac{J}{2} = \pi R^3 t$$



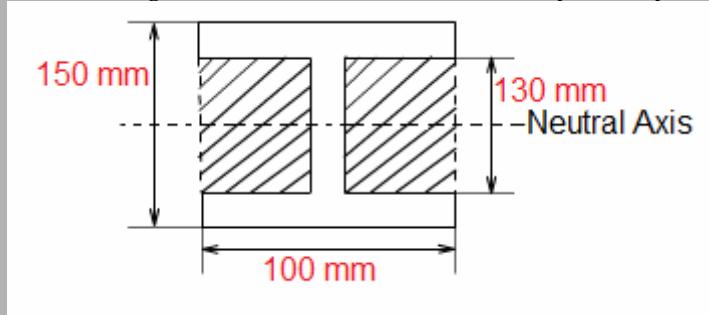
#### (v) Moment of inertia of a elliptical area

$$I_{xx} = \frac{\pi a b^3}{4}$$



**Let us take an example:** An I-section beam of 100 mm wide, 150 mm depth flange and web of thickness 20 mm is used in a structure of length 5 m. Determine the Moment of Inertia (of area) of cross-section of the beam.

**Answer:** Carefully observe the figure below. It has sections with symmetry about the neutral axis.



We may use standard value for a rectangle about an axis passes through centroid. i.e.  $I = \frac{bh^3}{12}$ .

The section can thus be divided into convenient rectangles for each of which the neutral axis passes

$$\begin{aligned} I_{\text{Beam}} &= I_{\text{Rectangle}} - I_{\text{Shaded area}} \\ \text{the centroid.} &= \left[ \frac{0.100 \times (0.150)^3}{12} - 2 \times \frac{0.40 \times 0.130^3}{12} \right] \text{m}^4 \\ &= 1.183 \times 10^{-4} \text{ m}^4 \end{aligned}$$

### 3.9 Radius of gyration

Consider area  $A$  with moment of inertia  $I_{xx}$ . Imagine that the area is concentrated in a thin strip parallel to

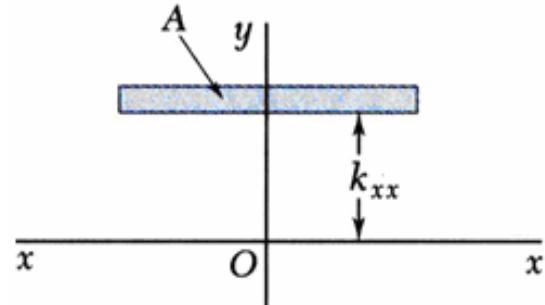
### Chapter-3

### Moment of Inertia and Centroid

the  $x$  axis with equivalent  $I_{xx}$ .

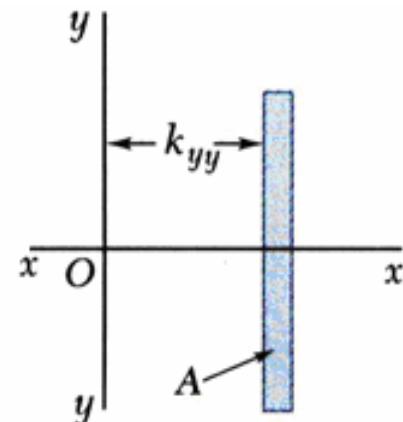
$$I_{xx} = k_{xx}^2 A \text{ or } k_{xx} = \sqrt{\frac{I_{xx}}{A}}$$

$k_{xx}$  = radius of gyration with respect to the  $x$  axis.



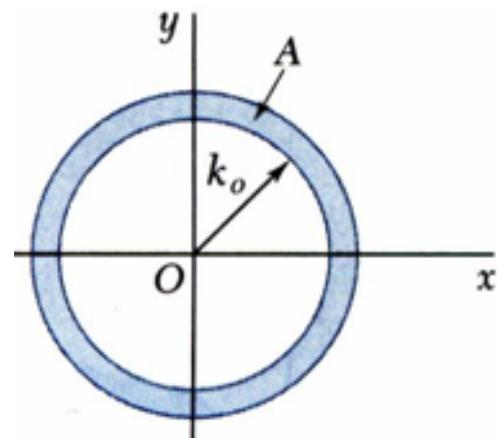
Similarly

$$I_{yy} = k_{yy}^2 A \text{ or } k_{yy} = \sqrt{\frac{I_{yy}}{A}}$$



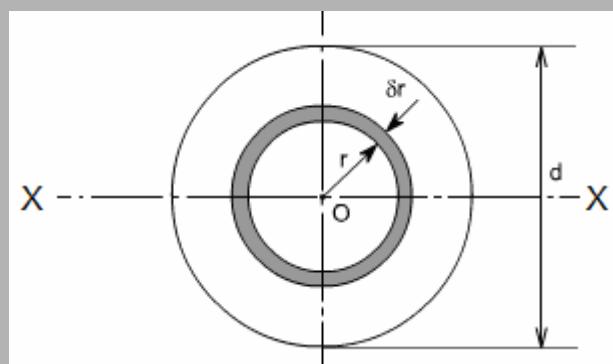
$$J = k_o^2 A \text{ or } k_o = \sqrt{\frac{J}{A}}$$

$$k_o^2 = k_{xx}^2 + k_{yy}^2$$



**Let us take an example:** Find radius of gyration for a circular area of diameter 'd' about central axis.

**Answer:**



We know that,  $I_{xx} = k_{xx}^2 A$

$$\text{or } K_{xx} = \sqrt{\frac{I_{xx}}{A}} = \sqrt{\frac{\pi d^4}{\frac{64}{4}}} = \frac{d}{4}$$

## OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

#### Moment of Inertia (Second moment of an area)

GATE-1. The second moment of a circular area about the diameter is given by (D is the diameter) [GATE-2003]

(a)  $\frac{\pi D^4}{4}$       (b)  $\frac{\pi D^4}{16}$       (c)  $\frac{\pi D^4}{32}$       (d)  $\frac{\pi D^4}{64}$

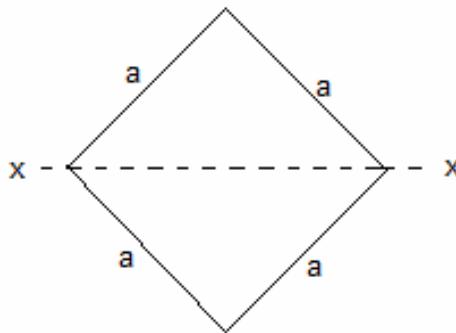
GATE-1. Ans. (d)

GATE-2. The area moment of inertia of a square of size 1 unit about its diagonal is:

[GATE-2001]

(a)  $\frac{1}{3}$       (b)  $\frac{1}{4}$       (c)  $\frac{1}{12}$       (d)  $\frac{1}{6}$

GATE-2. Ans. (c)  $I_{xx} = \frac{a^4}{12} = \frac{(1)^4}{12}$

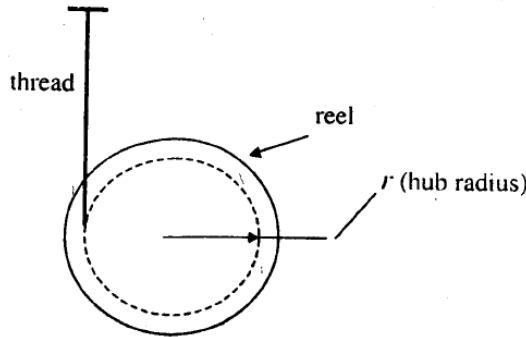


#### Radius of Gyration

Data for Q3–Q4 are given below. Solve the problems and choose correct answers.

A reel of mass “m” and radius of gyration “k” is rolling down smoothly from rest with one end of the thread wound on it held in the ceiling as depicted in the figure. Consider the thickness of the thread and its mass negligible in comparison with the radius “r” of the hub and the reel mass “m”. Symbol “g” represents the acceleration due to gravity.

[GATE-2003]

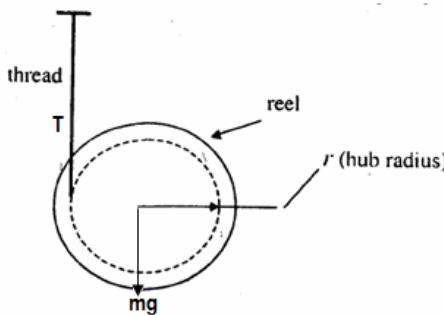


**GATE-3.** The linear acceleration of the reel is:

(a)  $\frac{gr^2}{(r^2 + k^2)}$       (b)  $\frac{gk^2}{(r^2 + k^2)}$       (c)  $\frac{grk}{(r^2 + k^2)}$       (d)  $\frac{mgr^2}{(r^2 + k^2)}$

**GATE-3. Ans. (a)** For downward linear motion  $mg - T = mf$ , where  $f =$  linear tangential acceleration  $= ra$ ,  $\alpha =$  rotational acceleration. Considering rotational motion  $Tr = I\alpha$ .

$$\text{or, } T = mk^2 \times \frac{f}{r^2} \text{ therefore } mg - T = mf \text{ gives } f = \frac{gr^2}{(r^2 + k^2)}$$



**GATE-4.** The tension in the thread is:

(a)  $\frac{mgr^2}{(r^2 + k^2)}$       (b)  $\frac{mgrk}{(r^2 + k^2)}$       (c)  $\frac{mgk^2}{(r^2 + k^2)}$       (d)  $\frac{mg}{(r^2 + k^2)}$

**GATE-4. Ans. (c)**  $T = mk^2 \times \frac{f}{r^2} = mk^2 \times \frac{gr^2}{r^2(r^2 + k^2)} = \frac{mgk^2}{(r^2 + k^2)}$

## Previous 20-Years IES Questions

### Centroid

**IES-1.** Assertion (A): Inertia force always acts through the centroid of the body and is directed opposite to the acceleration of the centroid. [IES-2001]

Reason (R): It has always a tendency to retard the motion.

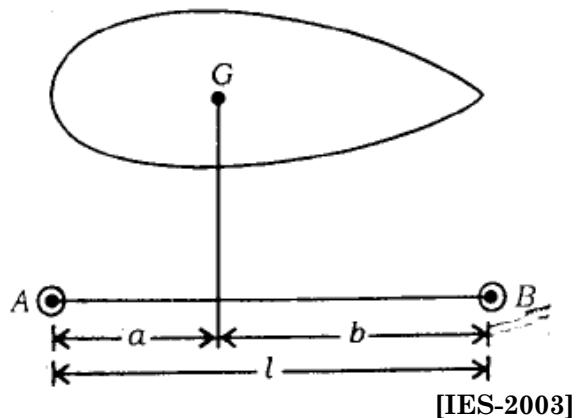
- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-1. Ans. (c)** It has always a tendency to oppose the motion not retard. If we want to retard a motion then it will want to accelerate.

## Radius of Gyration

IES-2. Figure shows a rigid body of mass  $m$  having radius of gyration  $k$  about its centre of gravity. It is to be replaced by an equivalent dynamical system of two masses placed at A and B. The mass at A should be:

(a)  $\frac{a \times m}{a+b}$       (b)  $\frac{b \times m}{a+b}$   
 (c)  $\frac{m}{3} \times \frac{a}{b}$       (d)  $\frac{m}{2} \times \frac{b}{a}$



IES-2. Ans. (b)

IES-3. Force required to accelerate a cylindrical body which rolls without slipping on a horizontal plane (mass of cylindrical body is  $m$ , radius of the cylindrical surface in contact with plane is  $r$ , radius of gyration of body is  $k$  and acceleration of the body is  $a$ ) is: [IES-2001]

(a)  $m(k^2/r^2 + 1).a$       (b)  $(mk^2/r^2).a$       (c)  $mk^2.a$       (d)  $(mk^2/r + 1).a$

IES-3. Ans. (a)

IES-4. A body of mass  $m$  and radius of gyration  $k$  is to be replaced by two masses  $m_1$  and  $m_2$  located at distances  $h_1$  and  $h_2$  from the CG of the original body. An equivalent dynamic system will result, if [IES-2001]

(a)  $h_1 + h_2 = k$       (b)  $h_1^2 + h_2^2 = k^2$       (c)  $h_1 h_2 = k^2$       (d)  $\sqrt{h_1 h_2} = k^2$

IES-4. Ans. (c)

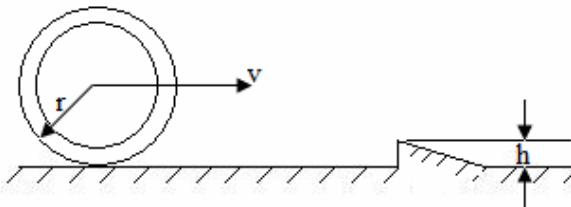
## Previous 20-Years IAS Questions

### Radius of Gyration

IAS-1. A wheel of centroidal radius of gyration 'k' is rolling on a horizontal surface with constant velocity. It comes across an obstruction of height 'h'. Because of its rolling speed, it just overcomes the obstruction. To determine  $v$ , one should use the principle (s) of conservation of [IAS 1994]

(a) Energy      (b) Linear momentum  
 (c) Energy and linear momentum      (d) Energy and angular momentum

IAS-1. Ans. (a)



## Previous Conventional Questions with Answers

### ***Conventional Question IES-2004***

**Question:** When are I-sections preferred in engineering applications? Elaborate your answer.

**Answer:** I-section has large section modulus. It will reduce the stresses induced in the material. Since I-section has the considerable area are far away from the natural so its section modulus increased.